MATHEMATICAL SCIENCES RESEARCH INSTITUTE

EMISSARY

FALL 2001

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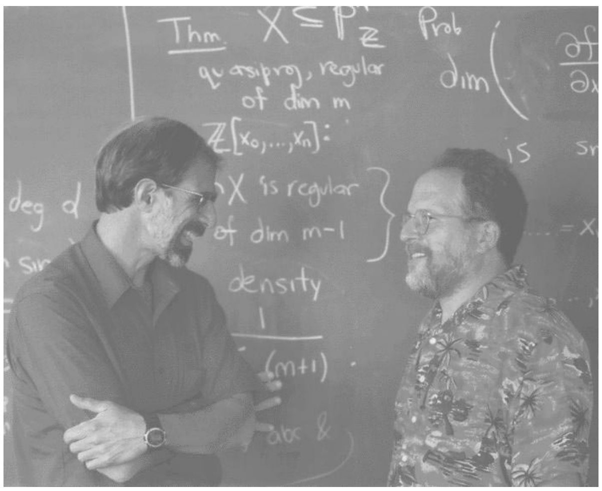
Notes from the Director

David Eisenbud



David Eisenbud and MSRI's new Deputy Director, Michael Singer (see page 19, right)

MSRI is humming along as usual at this time of year. Our two big programs share a lot: Integral Geometry is in many ways the pure side of Inverse Problems. The interaction of these two groups started off with a joint introductory workshop, and continues to be very lively indeed. You can read a tiny vignette from integral geometry in the first posting on our "Concept of the Month" page (www.msri.org/publications/cotm/). Each posting will be a one-page description by an expert, but at a level that should be accessible to beginning graduate students, of some basic notion from one of our current areas. The first, by Goncharov, is on Crofton's formula from integral geometry. Comments and suggestions for future "Concepts" are welcome!

I'm now starting my fifth year at MSRI — probably no one finds this as surprising as I do. I've tremendously enjoyed the excitement of this place, and the many possibilities it affords. Of course these possibilities would be purely theoretical without the

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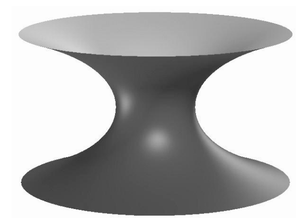
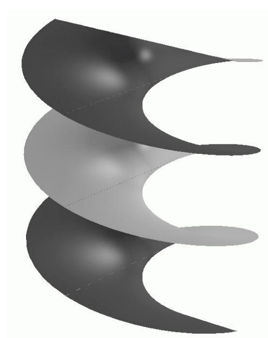
Clay Mathematics Institute Summer School on the Global Theory of Minimal Surfaces

David Hoffman

For five weeks this summer, the Institute was the world center for the study of minimal surfaces. MSRI hosted the Clay Mathematics Institute Summer School on the Global Theory of Minimal Surfaces, during which 150 mathematicians — undergraduates, post-doctoral students, young researchers, and the world's experts — participated in the most extensive meeting ever held on the subject in its 250-year history.



In 1744, Euler posed and solved the problem of finding the surfaces of rotation that minimize area. The only answer is the catenoid, shown to the right. Some eleven years later, in a series of letters to Euler, Lagrange, at age 19, discussed the problem of finding a graph over a region in the plane, with prescribed boundary values, that was a critical point for the area function. He wrote down what would now be called the Euler–Lagrange equation for the solution (a second-order, nonlinear and elliptic equation), but he did not provide any new solutions. In 1776, Meusnier showed that the helicoid (shown on the left) was also a solution. Equally important, he gave a geometric interpretation of the Euler–Lagrange equation in terms of the vanishing of the average of the principal curvatures of the surface, a quantity now known, after a suggestion of Sophie Germain, as the mean curvature. (This short outline of the earliest history of the subject is based on the introduction to J. C. C. Nitsche's *Lectures on Minimal Surfaces*, published by Cambridge University Press.



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good ideas that pour in from you, our mathematical community, and without the work of the organizers and staff here, who make these projects into reality. My original term was for five years, but I both enjoy and value the activity of MSRI so much that I've accepted a second term. I look forward to seeing many of you here over the next five years as well!

Summer Research at MSRI

With the Banff Center starting up (see below), we're planning to have a few less workshops each year at MSRI. One benefit is that the summer will be freed up for people to come and work here singly or in small groups. This should be particularly useful for the NSF supported Focused Research Groups, but the program is certainly not limited to those.

This year we'll host individuals or small groups for a minimum of two weeks between June 3, 2002 and July 31, 2002. You'll get office space and access to our computers and library so that you can work on your research projects. Support this summer will be quite limited, but we hope to expand the program in future years. To find out how to apply to come, see the link on our homepage.

Berkeley is particularly nice in the summer – give it a try!

Banff

MSRI has entered into a partnership with the Pacific Institute for the Mathematical Sciences with the goal of providing for North America a research center analogous to the wonderful institutes in Oberwolfach and Luminy: The *Banff International Research Station* (BIRS). The Station is intended to provide a home for conferences ranging across pure and applied mathematics and their uses in the sciences, engineering, and industry. It will also operate in other modes: for instance, it will be possible to come to Banff to spend time with small groups of colleagues in the "Research In Teams" mode (modeled on Oberwolfach's Research in Pairs). An "Aspen Mode" will provide a less formal structure for somewhat larger groups to assemble in the summers. I urge you to look at the website (go to www.msri.org and click on the Banff icon) — it contains a "call for proposals" and lots of details. The first round of proposals was due October 21, 2001, too late for those learning about Banff from this newsletter; never mind, it's time to gear up to put in a proposal next year!

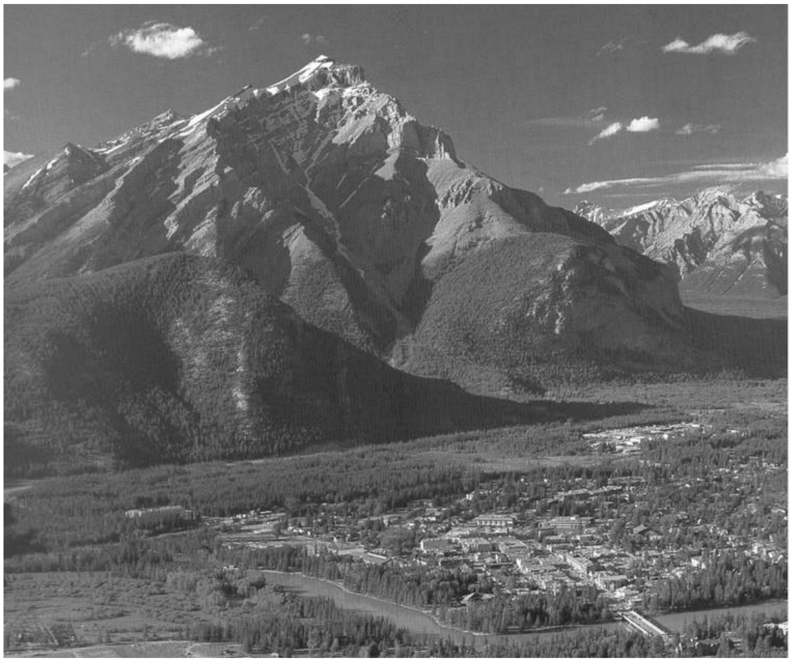
The setting, like MSRI's own, is spectacular: Banff is a huge national park in the Canadian Rockies in a valley with high, snow-capped mountains encircling it and excellent hiking and skiing very nearby.

The Banff Centre is interesting in itself: a campus devoted to the arts, including large facilities for music, dance, theater, television, and even a group working on computers and art. It's part of the University of Calgary and has many amenities that one would expect of a university campus, such as an athletic facility, dining hall, and pub. The Centre will renovate to our specifications a dormitory building for 40 researchers and a lecture/computer/discussion area, so it should serve the needs of mathematical presentation and collaboration. It will be an extremely flexible setting: since the Centre runs a major conference facility with about 400 rooms, mathematical meetings could expand into this space when necessary.

The benefits of Banff will be many. Aside from the breadth of the proposed activities and the relief of pressure on some of the institutes (especially MSRI!) for conferences, there is a particular benefit to young mathematicians which may not be so obvious. Though senior and established US mathematicians are often invited to the centers in Oberwolfach and Luminy, those insititutes can serve their local German and French populations in a way that they cannot serve North Americans: European graduate students and postdocs are often present at Oberwolfach and at Luminy, and this gives them a fantastic opportunity both to meet their seniors and their future colleagues and to experience the excitement of a group of high-level mathematicians directly.

I have seen this in action many times in my visits, but I have a personal appreciation as well: through rather exceptional luck I was invited to an Oberwolfach event just a year after my Ph.D. The excitement of lecturing there was extraordinary – I still remember the moment vividly. The attractiveness of a mathematical career was never clearer to me! But most important, the young German mathematicians I met there have remained my collaborators and friends to this day, and formed the basis for a network of European contacts that has been extremely important to me in my work ever since. I can truly say that the invitation changed my life.

Please turn to page 12 to see how we're strengthening the Institute through private fundraising — and join me in supporting MSRI!



Spectral Invariants

Rafe Mazzeo

Spectral Invariants, one of the two programs at MSRI in the Spring of 2001, is a field loosely encompassing a number of areas of study which have grown out of the basic problems of spectral geometry, suggested by Mark Kac's (too often quoted) question: can one hear the shape of a drum? The term 'spectral invariants' was used to emphasize the fact that the unifying feature of the program would be the study of quantities which originally arose in spectral geometry (or index theory) but which have since taken on a life of their own.

To describe some of these developments, first recall that if A is a symmetric n-by-n matrix (with real entries), then its spectrum consists of its set of eigenvalues, $\{\lambda_1, \ldots, \lambda_n\}$. These are listed in nondecreasing order and with multiplicity, so there are always exactly n of them. Conversely, this n-tuple determines the matrix A, amongst symmetric matrices, up to conjugacy by an orthogonal matrix. Now suppose (M, g) is an n-dimensional closed Riemannian manifold. Its Laplace-Beltrami operator Δ_g is a self-adjoint, nonnegative elliptic operator acting on $L^2(M, dV_g)$, the space of square-integrable functions. It is classical that this operator may be diagonalized, but now there are an infinite number of eigenvalues. Listing these again in nondecreasing order, with multiplicity, we obtain an infinite list of nonnegative real numbers $\{0 = \lambda_0, \lambda_1, \ldots\}$, which is called the spectrum of Δ_g. Loosely speaking, the field of spectral geometry is concerned with the precise relationships between this spectrum of Δ_g and the geometry of the Riemannian manifold (M, g).

There is no reason why one should only look at the spectrum of the scalar Laplacian, and it is also of interest to seek similar relationships for the Hodge Laplacian, acting on differential forms, or the Dirac (or Dirac-type) operators. More generally still, the spectral analysis of (stationary) Schrödinger operators of the form $\Delta + V$ on $\mathbb{R}^n$ is central in some parts of mathematical physics. For simplicity and brevity, we shall mostly focus on the spectrum of the scalar Laplacian here. However, we also include the case where M is a manifold with boundary (e.g. a domain in $\mathbb{R}^n$), and Δ_g acts on functions which satisfy Dirichlet boundary conditions, i.e. vanish at the boundary.

The study of spec (Δ_g) naturally divides into two parts, one focusing on the geometric information encoded in the 'low eigenvalues' and the other on the high frequency asymptotics and other properties concerning the statistical distribution of the sequence of eigenvalues. In the first, one seeks to find both upper and lower estimates for the first few eigenvalues in terms of geometric quantities on the manifold. Estimates from above on an eigenvalue are not hard to come by, since eigenvalues may be described by a minimax procedure from the classical Rayleigh quotient $\int |\nabla u|^2 dV_g / \int |u|^2 dV_g$. However, lower estimates are typically much more subtle; one example is the lower bound discovered by Cheeger for the first nonzero eigenvalue λ_1 in terms of an isoperimetric ratio. Geometric estimates on higher eigenvalues become progressively harder to obtain.

The basic result concerning the so-called 'high frequency' asymptotics of the spectrum is the Weyl estimate, concerning the counting function $N(\lambda)$, the number of eigenvalues less than or equal to λ. This states that $N(\lambda)$ grows as $c_n \mathrm{Vol}(M)\lambda^{n/2}$, where c_n is a purely dimensional constant. Although this result in its simplest form does indeed trace back to Weyl, more recent developments (since the 1960's) revolve around optimal estimates for the remainder $N(\lambda) - c_n \mathrm{Vol}(M)\lambda^{n/2}$. For the Laplacian, these are due to Avakumovic (and later by Hörmander for more general pseudodifferential operators, in his famous paper introducing the theory of Fourier Integral Operators); Duistermaat and Guillemin and then Ivrii showed how to obtain the sharpest form. One of the beautiful discoveries here is that the dynamics of the geodesic flow on M is closely related to the size of this remainder term. The manifolds for which this remainder is the largest are those such as the sphere with all geodesics closed, while it is quantitatively smaller when the set of closed geodesics has measure zero, in an appropriate sense.

Which nondecreasing lists of numbers can arise as the spectrum of Δ_g for some manifold M and some metric g? Quite remarkably, up to some very mild conditions, any finite list $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$ can occur as the first N terms of the spectrum of a Laplace operator for some manifold and some metric. This surprising fact, discovered by Colin de Verdiere, is proved by first establishing the analogous fact for the combinatorial Laplacian on some finite graph, and then considering the manifold obtained by 'thickening up' this graph. In contrast, although the Weyl asymptotic estimate is an obvious constraint on the tail of the sequence, it is far from the only one as we shall see shortly, and it is most likely a hopeless task to characterize the range of this 'spectrum mapping'.

The uniqueness question, concerning how many Riemannian manifolds can have the same spectrum, has also been much studied. At this point there are only a rather small number of examples of manifolds or domains which are know to be 'spectrally determined', i.e. uniquely determined by their spectrum. In particular, Zelditch has established the spectral determination of a large class of real analytic domains in the plane with a single reflection symmetry; these results were announced at the March workshop for this spectral invariants program. On the other hand, there are many counterexamples, and many general constructions which give rise to pairs (or larger families) of 'isospectral manifolds'.

To study each of these questions, it has proved to be extremely fruitful to consider, rather than individual eigenvalues, functions which are built out of the full spectrum. There are three of greatest interest: the heat trace $H(t) = \sum \exp(-\lambda_j t)$, the wave trace $W(t) = \sum \exp(\pm i\sqrt{\lambda_j} t)$ and the zeta function $\zeta(s) = \sum \lambda_j^{-s}$. The first two of these get their names from the fact that they are the (Hilbert space operator-theoretic) traces of the fundamental solutions of the heat and wave equations, respectively, corresponding to the Laplacian. The third is a trace too, of the (complex) powers of the Laplacian. $H(t)$ is defined and smooth on $\mathbb{R}^+$, and its geometric information is contained in the coefficients of its asymptotic expansion as $t \searrow 0$. These coefficients are called the heat invariants of the metric. $W(t)$ is defined as a distribution on the whole line, and it has a singularity at $t = 0$, but also, quite

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BMC and BAMO

Joe Buhler and Zvezdelina Stankova

On Sundays from mid-September through mid-May twenty to sixty high school students gather on the U.C. Berkeley campus to listen to, talk about, and work on advanced mathematical topics. Real dedication is required on the part of the students — for instance, students coming from Palo Alto, San Jose, or Davis to the Berkeley Math Circle give up as much as 5 to 7 hours of precious weekend time to attend. In addition, hard work is also required from organizers, teachers, and parents (e.g., parking near campus is non-trivial even on Sundays).

The idea for the “Math Circles” originated in Hungary more than a century ago, and gradually spread throughout the world, finally making inroads in a few places in the United States in recent years. The Circles are intended for talented middle and high school math students, and the material goes well beyond the typical high school curriculum. The focus is on the joy of working on challenging problems and exploring novel mathematical ideas.

Math Circles led to national and international math competitions. The International Mathematical Olympiad (IMO) started in 1959 in Romania and was intended only for Eastern European block countries but has now grown to include more than 80 countries from all over the world. The United States joined in 1974 and hosted the Olympiad in 1981 and 2001.

As a highlight of the Berkeley Math Circle (BMC) year, a local math competition called the Bay Area Mathematical Olympiad (BAMO) started in 1999. Every year, about 250 students from over 40 schools take the exam. To get a flavor of the problems posed, see the sidebar. (They have been edited for the Emissary; the original version and solutions can be found at http://mathcircle.berkeley.edu/BAMO2001.html.)

The BMC and BAMO have become fixtures in the high school math scene in the Bay Area. They originally resulted from organizational efforts of Hugo Rossi (then Deputy Director of MSRI), Zvezdelina Stankova (then a postdoc at MSRI), and Paul Zeitz (on the math faculty at the University of San Francisco). Stankova and Zeitz had considerable experience training middle and high school students for national and international math competitions, as former participants in the IMO on the Bulgarian and US teams, respectively, and coaches of the US national math team for the IMO. Stankova undertook the difficult task of organizing and running the Berkeley Math Circle.

After spending the 1997–98 academic year popularizing the idea, and raising money to support it, the BMC and BAMO were begun in 98–99. As part of the lead-in to the initial year, high school teachers were invited to MSRI for a “Conversation.” A streaming video lecture announcing these events can be found on the MSRI web site at www.msri.org/activities/events/9798/t2t/index.html. Math circles were also founded in 1998 by Tom Rike at Oakland High School, by Tatiana Shubin at San Jose State University, and by Tom Davis and Joshua Zucker in Palo Alto.

In subsequent years the BMC and BAMO have been supported by MSRI, Mill College, San Jose State University, the University of California at Berkeley, the University of San Francisco, the American Institute of Mathematics, and private donations. In addition we have received generous grants from the Packard Foundation for the academic years 2000-01 and 2001-02, and from the Clay Mathematics Institute for 2001-02.

The math competitions generate considerable excitement among the students, and the depth of the impact can be seen in many ways. One is the carryover of involvement in the BMC and BAMO here to the International Math Olympiad. In the 2001 IMO, held in Washington, D.C., three of the six U.S. team members were from the Berkeley Math Circle. Gabriel Carroll wrote a perfect paper (both he and Tiankai Liu received gold medals), and Oaz Nir received a silver medal. Gabriel will be going to Harvard next year, Oaz will be going to Duke; and Tiankai, as a freshman, is eligible to take the IMO exam for three more years. Andrew Wiles, Edward Witten, and Rita Colwell (to name a few math/science celebrities) personally presented the gold medals at the IMO’01.

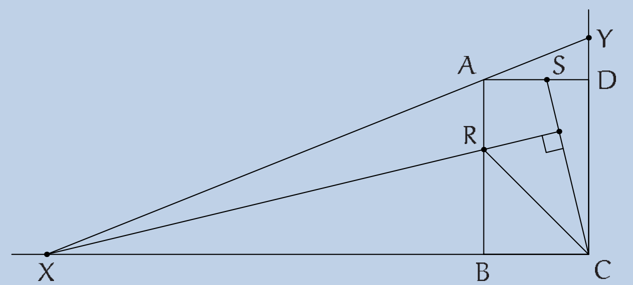
Gabriel Carroll has been in the news quite a bit recently, since he also finished in 3rd place in the Intel Science Competition, pointing to Stankova in his application as the person who had most impact on his mathematical career.

In Eastern Europe, university faculty are often in direct contact with bright young high school students through the mathematical circles. In the USA, unfortunately, few mathematicians have the willingness or the time to spend communicating the beauty of mathematics to teenagers. The Berkeley Math Circle and BAMO are exceptions, and MSRI has been instrumental in fostering those activities, bringing talented high school math students into contact with practicing mathematicians.

For more information on Math Circles and how to start or support one, see http://mathcircle.berkeley.edu.

1. Each vertex of a convex plane polygon is colored red, white, or blue in such a way that all colors are used and no two adjacent vertices have the same color. Prove that the polygon can be triangulated (without adding new vertices) in such a way that all triangles have a vertex of each color.

2. Let $ABCD$ be a rectangle, and let R and S be points on the sides AB and AD respectively. Let X be the intersection of the line BC and the line through R that is perpendicular to CS, and let Y be the intersection of the line CD with the line through S that is perpendicular to CR. Show that A, X, and Y are collinear.



3. Suppose that f is a function mapping positive integers to positive integers, satisfying $f(n+1) > f(n)$ and $f(f(n)) = 3n$ for all positive integers n. Find $f(2001)$.

She Grapples With Operations Research

Sandra M. Gilbert

... the by now famous problem of the jeep ... concerns a jeep which is able to carry enough fuel to travel a distance d*, but is required to cross a desert whose distance is greater than* d *(for example* $2d$*). It is to do this by carrying fuel from its home base and establishing fuel depots at various points along its route so that it can refuel as it moves further out. ... [but] in general, the more jeeps one sends across, the lower fuel consumption per jeep.*

– David Gale, "The Jeep Once More or Jeeper by the Dozen"

The mathematician is crossing the desert,
his fine high features creased with thought.

One tank of fuel at this depot, another stashed at that.
How many caches needed in between?

She worries. It's all too Zeno for her liking.
And what if he insists on the Sahara?

No, he promises, he'll only try the kindlier
Mojave this time, with its rainstruck buds and rare

new blossoms rising while his jeep,
his squad of jeeps, moves slowly on the trip

through sand, through quarks and quirks of sand,
their particles an endless series

as she waits and hates his danger.
The mathematician crosses, curses, blesses

the infinite regressions of the desert:
and the desert sun storms down like thunder, like a roar

of light against his beard, his temples
clenched with calculations

and desire.
At stated stations,

palms, dates, springs of comfort
will appear. And there he'll prudently

sequester further energies.
Blank sky and melting gold, keen blade

of lemmas roaring through his engine.
She stands on the sidelines in the shade.

She stirs a pitcher of gin and lemonade.
Astute, her body manufactures

leafy murmurs as she turns herself
into a crystal dish of peaches.

The mathematician is crossing the desert,
crossing, journeying past Zeno, past the infinite.

She wants to be the first
oasis that he reaches.

This poem and the ones on pages 7 and 13 are taken from poet and literary critic Sandra Gilbert's *Kissing the Bread: New and Selected Poems, 1969–1999* (Norton, 2000). They appear in the section titled "When She Was Kissed by the Mathematician".

Spectral Invariants

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beautifully, at (some subset of) the lengths of the closed geodesics on the manifold M. This famous theorem was proved by Chazarain and in sharper form by Duistermaat-Guillemin. (That this 'length spectrum' is a spectral invariant was obtained somewhat earlier by Colin de Verdiere using the heat equation continued to complex time.) The asymptotic expansion of $W(t)$ as $t \to 0$ contains the same information as the expansion of $H(t)$, but its expansions at these other singularities contain new information, related to the dynamical properties (in particular, the Birkhoff normal form) of the geodesic flow near the corresponding closed geodesics, and is very difficult to obtain. Only in the past few years have some of the secrets of these nonlocal invariants been unlocked, in work of Guillemin and of Zelditch. Finally, using the Weyl asymptotics it is clear that $\zeta(s)$ is holomorphic in the half-plane $\mathrm{Re}\,(s) > n/2$, but it turns out to extend meromorphically to the entire complex plane. Numerical invariants are obtained by evaluating $\zeta(s)$ at special points. Of greatest interest is the value $-\zeta'(0)$, which was introduced by Ray and Singer as a viable way to define the logarithm of the determinant of Δ_g.

Each of these functions determines the other two by various of the classical transforms, and thus the geometric information carried in any one of these functions is somehow latent in the other two. However, some types of information may be easier to obtain from one than the other. For example, to derive the expansions for $H(t)$ and $W(t)$, one makes direct use of the associated underlying PDE's (the heat and wave equations) and constructs approximate 'parametrices' to the fundamental solutions for these equations using local geometric methods. The parametrix for the heat equation is obtained by quite elementary means, and provides an easy link between the heat invariants and local geometric data. (Pursued further, this leads to the heat equation proof of the Atiyah-Singer index theorem.) On the other hand, the spectral invariants associated to closed geodesics are most naturally obtained using the wave equation, for which a parametrix must be constructed using Fourier integral operator techniques.

These functions all have classical antecedents: theta functions, the Poisson summation formula, and number-theoretic zeta functions, respectively. Just as for these arithmetic analogues, the location of singularities and existence of expansions for $H(t)$ and $W(t)$ and the meromorphy of the zeta function are indicative of some of the concealed structure of the eigenvalue sequence.

These functions are the main examples of spectral invariants, and as indicated earlier, the body of results which have emerged from their study goes considerably beyond the original scope of spectral geometry.

I shall describe just a few themes of some of these developments. First, one remarkable and useful feature of the zeta function, and particularly the determinant, is the existence of 'cut and paste' formulas, which allow some way of partially localizing this global invariant into contributions from the pieces of a decomposition

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Spectral Invariants

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of the manifold. This 'Mayer-Vietoris principle' was explored in great detail by Burghelea, Friedlander and Kappeler and by Hassell and others, and it plays a significant role in some proofs of the Cheeger-Müller theorem on the equality of analytic and Reidemeister torsions. This is a long story, worthy of a separate account, so I turn instead to the well-known results on compactness of isospectral sets of metrics. These give some control on the uniqueness question and are perhaps the best general results in this direction that could be expected. The basic point is that sets of metrics which share the same spectrum are shown to be compact in an appropriate topology by demonstrating that the eigenvalues, or some of these spectral invariants, control the geometry of the metrics in a strong way. Melrose made the original observation (during his participation in the program in MSRI's first year of operation, 1982-3) that for planar domains, the heat invariants serve essentially as a sequence of nonlinear Sobolev norms of the curvature function of the boundary; since these are all constant on an isospectral set, the curvature functions must lie in a compact set. (A related result had been obtained earlier by Brüning in a somewhat simpler context.) Later, Osgood, Phillips and Sarnak proved that isospectral sets of metrics on Riemann surfaces are compact. To do this they used $\log\det\Delta_g$ as a 'height function' on the space of metrics; they showed moreover that within any conformal class this function attains its unique minimum precisely at the constant curvature metric. This result introduces an important theme: that canonical metrics are closely related to (first variations of) spectral invariants. The key tool here is a formula, coming out of string theory, and due to Polyakov, which computes the variation of this determinant of the Laplacian with respect to any one-parameter family of conformally related metrics. The determinant is a global invariant of the metric, but quite remarkably, this variation turns out to be given by a local expression, i.e. an integral over the surface involving the conformal factor and explicit geometric quantities.

The existence of this Polyakov variational formula is closely tied to the fact that the Laplacian in two dimensions behaves quite simply under conformal changes of metric. Formulas of this type exist in greater generality for natural geometric operators which exhibit a conformal covariance. A naturally geometric operator is one which is associated canonically to a Riemannian metric; this operator is conformally covariant if the two operators corresponding to two conformally related metrics are simply related by pre- and post-multiplication by powers of the conformal factor. The Laplacian in dimensions greater than two does not have this property, but for example, adding a zeroeth order correction term involving the scalar curvature produces a conformally covariant operator known as the conformal Laplacian.

This suggests one line of questions: Can one characterize the conformally covariant operators in general? Are there good compactness results for isospectral families (within a conformal class) for these? Is there any hope for obtaining similar results for non conformally covariant operators?

To address this last question first, there has been some success in obtaining compactness for sets of conformally related isospectral metrics for the Laplacian in dimensions three and four, by the work of Chang, Yang, Gursky and others. In addition, Okikiolu has shown how to analyze the variation of the determinant of the Laplacian across conformal classes and how canonical metrics, e.g., round metrics on spheres, are often extremals for this function.

Next, what about the existence of other conformally covariant operators, beyond the classical example of the conformal Laplacian, and some newer ones, such as the fourth order Paneitz operator? Within the last several years, substantial progress has been made on understanding when such operators exist and when they do not,

The generalized Poisson summation formula

The fundamental identity

$$\sum_{n=-\infty}^{\infty} e^{inx} = 2\pi \sum_{n=-\infty}^{\infty} \delta(x - 2\pi n)$$

is well-known in Fourier analysis as the Poisson summation formula. It is equivalent to the Fourier inversion formula for periodic functions on the line. The summands on the right are Dirac delta functions centered at the points $2\pi n$, $n \in \mathbb{Z}$, while the sum on the left is the Fourier transform of the sum of Dirac delta functions centered at points points in the integer lattice $\mathbb{Z} \subset \mathbb{R}$. The sum on the left does not converge in any classical sense, and accordingly, the object on the right is only a generalized function, i.e a distribution.

There is a very beautiful generalization of this formula on general closed Riemannian manifolds (M, g). If the Laplace-Beltrami operator Δ_g has eigenvalues λ_n^2, $n = 0, 1, 2, \ldots$, then we define the *wave trace* of Δ_g to be

$$W(t) = \sum_{n=0}^{\infty} e^{\pm i\lambda_n t}.$$

(For each n there are two terms in this sum, one of each sign.) This is the Hilbert space trace of the fundamental solution of the wave equation associated to Δ_g. Again, this sum converges only as a distribution. We also define the *length spectrum* of (M, g) to be the collection $\{L_\gamma\}$ of lengths of (smooth) closed geodesics on this manifold, with multiple coverings of the same geodesic allowed. It is unreasonable to expect any sort of closed form expression for $W(t)$, but the general Poisson summation formula states that $W(t)$ is actually a smooth function for most values of t. Its singularities are contained within the set $\{-L_\gamma\}\cup\{0\}\cup\{L_\gamma\}$; for generic metrics g, the singularities of $W(t)$ coincide precisely with this set. (Unfortunately, for special metrics, e.g. those with lots of symmetry, cancellations in the sum may cause some of these singularities to disappear.)

When (M, g) is the circle of length 2π, then its Laplacian is $-d^2/(dx)^2$, which has eigenvalues n^2, $n \in \mathbb{Z}$; on the other hand, the length spectrum of the circle is the set $\{2\pi n : n \in \mathbb{N}\}$.

This generalization was proved by Chazarain and then in sharper form by Guillemin and Duistermaat in the early 1970's.

through the work of Eastwood, Graham, and their collaborators. This subject has close ties to some parts of representation theory, and during this Spring's program, Graham and Zworski showed how some of these conformally covariant operators are closely related to the scattering theory of conformally compact Einstein metrics. (These metrics were originally introduced by Fefferman and Graham for the purpose of systematizing conformal invariant theory; to connect with some of the other focuses of this program, Anderson announced a very satisfactory existence theory for these metrics in four dimensions in the May workshop.)

Finally, Branson and Ørsted proved that the Polyakov formulæ for determinants of conformally covariant operators have a beautiful structure. In four dimensions, the variation of the (log of the) determinant, as always within a conformal class, is given by an expression of the form $\alpha \mathrm{I} + \beta \mathrm{II} + \gamma \mathrm{III}$, where I, II, and III are explicit functionals involving the conformal factor and local geometric quantities of the metric. Remarkably, these functionals are *universal*, by which I mean that different conformally covariant operators always lead to linear combinations of these same functionals; the coefficients α, β, γ alone depend on the specific operator!

Unlike the two-dimensional case, one does not hope for very simple characterizations of the extremals of these functionals, if for no other reason than that geometry in four dimensions is much more subtle. However, in the work of Chang, Yang, Gursky and their collaborators, some interesting geometric structure has arisen from the study of some of the individual functionals I, II, and III. In particular, an analysis of their extrema has led to some valuable geometric results, such as the existence of metrics with positive Ricci curvature in certain conformal classes. This direction of research is still in its infancy, and there remain many interesting open problems concerning these new classes of curvature equations coming from this variational theory and their geometric uses.

There are many other canonical metric constructions using determinants and their variations. Perhaps most famous is the Quillen metric on the determinant line bundle; Bismut and his collaborators have made an extensive and penetrating exploration of its geometry and have shown how to use it (and associated objects and constructions) in complex algebraic geometry. We also note the work of Müller and Wendland relating determinants to extremal Kähler metrics.

I have concentrated on just a few of the themes which were discussed during this Spring's program; most of the results mentioned above were due to mathematicians who took part for shorter or longer periods in this MSRI program.

I have of necessity omitted mention of many other important topics discussed during the seminars and workshops of this program, including the whole field of geometric scattering theory. To summarize this area very briefly: the spectral analysis of the Laplacian on complete noncompact Riemannian manifolds is fairly hopeless in general, but restricting to manifolds with some sort of regular structure at infinity (e.g., those which are asymptotically modelled at infinity by ends of locally symmetric manifolds), there is hope of developing a good theory. The correct set of objects to study, and a good set of directions to pursue (at least at first) is suggested by classical mathematical scattering theory (traditionally carried out for Schrödinger operators $\Delta + V$ on $\mathbb{R}^n$). The goal in this broader geometric context is to study the fine-scale structure of the spectrum, and other related objects and operators such as the scattering matrix and the resonances, and to establish relationships between these and the underlying geometry. New analytic tools must be developed, and the most successful of these come from Melrose's work on the geometrization of microlocal analysis. There is now a substantial methodology and body of results in this direction.

We have given here a hint of the flavor of this subject, or rather collection of interrelated subjects. The problems and directions of research discussed here constitute an important branch of geometric analysis, one which has deep connections and applications to many other fields. This Spectral Invariants semester provided a crucial forum for an appraisal of the current state of this subject.

They Debate Triangles and Medians

Sandra M. Gilbert

That they are is obvious to him,
remarkable to her.
She grants the points, their dark

necessity, each a moment brimming
with its own being–
and the lines, well, given points

and given time,
no doubt there must be lines,
those fateful journeyings

from here to there, from this to that.
But the vertices where journeys meet,
the angles, wide or narrow, yearning for closure

and then letting go–
aren't these, she asks, unlikely
as the medians that cling together

at the center of each triangle,
knotting altitudes and perpendiculars
into a single web of possibility?

And maybe Euclid got it halfway right:
in luminous sections, intersections,
everything is joined and rational,

at least for a while,
as if somebody had suddenly conjectured yes,
it can make sense–

and the triangles and medians
of you and me and them
last and glow till one by one

the fastenings unclasp
and that which must be linear
sheds the comforts of shape,

each line going its lonely distance
to the non-Euclidean place
where parallels diverge in darkness.

Puzzles Column

Joe P. Buhler and Elwyn R. Berlekamp

1. In a pitch-black room you are handed a deck of n cards, and are told that 10 of them are face up in the deck and the others are face down. You can manipulate the cards (say, to count them and determine n) but you cannot see the cards at all.

For which values of n can you separate the deck into two piles each of which is guaranteed to have the same number of face-up cards?

Remark: This is an old puzzle that recently received a burst of popularity on the internet after it was given as the weekly puzzle on the "Car Talk" NPR radio show.

2. A village has 24 voters, half Tories and half Whigs. All of them faithfully vote for their party's candidates. A court requires the village to be partitioned into 8 voting districts, each of which must consist of exactly three voters. Each district will then elect one representative to the village council. The details of the assignments of the 24 voters to the 8 districts are relegated to the current mayor of the village.

A naïve observer predicts that, since the two parties have equal numbers of voters, neither will be able to obtain a majority on the village council.

(a) Explain why the naïve observer is wrong.

(b) Make your own prediction: what fraction of the seats will be carried by the mayor's party?

3. A team of three plays the following game. After an initial session, during which they confer about what strategies they will follow, the teammates are put into separate hotel rooms. A judge flips a fair coin for each player and assigns that player a bit (0 or 1). Each player is then given the list of bits assigned to other players (matched to their names); however, players are not told what they themselves got. No communication is possible between the hotel rooms.

A. At this point, each player must send one of the following three statements to the judge: "my bit is 0", "my bit is 1", or "I pass". If at least one player doesn't pass and all non-pass statements are true, the team collectively shares a million-dollar prize. If all players pass or someone makes a false statement, they get nothing.

An obvious strategy gives the team a 50/50 chance to win: they agree that they will all pass except for one specific player, who will guess. Can you devise a strategy that gives the team a better chance of winning?

B. Image the same setup, but players are not allowed to pass: each must say "my bit is 0" or "my bit is 1". The team shares a million-dollar prize if and only if a majority of the statements are true. Find a good strategy for the team.

C. Same two puzzles as above for 7 people. Or for an arbitrary number (except that B requires that there be an odd number of people).

Comments: Puzzle A was discussed, in a different form, in an article in the April 10, 2001 Science Times section of the New York Times, written by Sara Robinson, who was then the MSRI journalist-in-residence (see also page 12). The puzzle illustrates the sometimes short distance between superficially recreational puzzles and research problems: when generalized to n people, an equivalent question arises in coding theory. The absolutely optimal strategy is known only for small values of n, and for n of the form $n = 2^k - 1$ and $n = 2^k$; the smallest value of n for which the optimal solution isn't known is $n = 9$.

Puzzle A first appeared in a different form in Todd Eberts' 1998 computer science Ph.D. thesis, and was converted to a problem about hats by Peter Winkler at Bell Labs; that was the form given in the New York Times article. The problem is given explicitly in terms of hats in a collection of problems that he is publishing in a forthcoming volume honoring Martin Gardner.

Puzzle B arises in a slightly different, but equivalent, form in the paper "The Expressive Power of Voting Polynomials" in 1993 by James Aspnes, Richard Beigel, Merrick Furst, and Steven Rudich.

Current Programs

See http://www.msri.org/calendar for details. See also page 18 for a list of workshops, including those belonging to these programs.

Integral Geometry, August 13, 2001 to December 14, 2001; organized by L. Barchini, S. Gindikin, A. Goncharov, and J. Wolf.

Inverse Problems, August 13, 2001 to December 14, 2001; organized by D. Colton, J. McLaughlin, W. Symes, and G. Uhlmann (chair).

Infinite-Dimensional Algebras and Mathematical Physics, January 7, 2002 to May 17, 2002; organized by E. Frenkel, V. Kac, I. Penkov, V. Serganova, and G. Zuckerman.

Algebraic Stacks, Intersection Theory, and Non-Abelian Hodge Theory, January 7, 2002 to May 17, 2002; organized by W. Fulton, L. Katzarkov, M. Kontsevich, Y. Manin, R. Pandharipande, T. Pantev, C. Simpson, and A. Vistoli.

MSRI Book Series

The series "Mathematical Sciences Research Publications" is published by Cambridge University Press. The contents are also available at http://www.msri.org/publications/books. The most recent volume (2001) is *Random Matrix Models and Their Applications*, edited by Pavel Bleher and Alexander Its, vol. 40, 438 p., ISBN 0521802091. Forthcoming:

Algorithmic Number Theory, edited by Joe Buhler and Peter Stevenhagen
New Directions in Hopf Algebras, edited by Susan Montgomery
More Games of No Chance, edited by Richard Nowakowski
Galois Groups, edited by Leila Schneps

Clay Summer School on Minimal Surfaces

(continued from page 1)

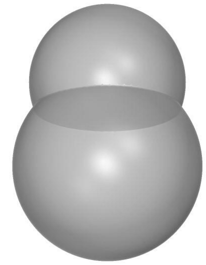
It was there that I read that while Meusnier refers to the geometric work of Euler, he does not mention Euler's discovery of the catenoid, publishing a derivation of the minimality of the catenoid himself. Lagrange, too, makes no mention of Euler in his published work on the subject. Of course, no statements or reflections on current practices in the field are intended or implied by the author's reporting of this historical curiosity.)

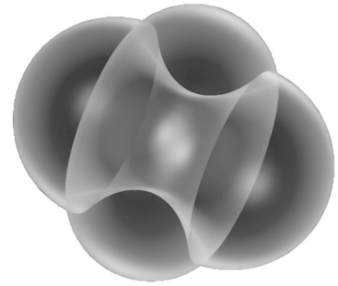
A *minimal surface* is one whose mean curvature is everywhere zero. It has the defining property that every sufficiently small piece of it (small enough, say, to be a graph over some plane) is the surface of least area among all surfaces with the same boundary. The simplest minimal surface is the flat plane, but other minimal surfaces are far from simple. They are realized in the physical world by soap films spanning closed curves, and they appear as interfaces where the pressure is the same on either side. Finding a surface of least area spanning a given contour is known as the Plateau Problem, after the nineteenth-century Belgian physicist Felix Plateau, who posed the problem of finding a mathematical description of these solutions (and, by implication, a proof that they exist).

Closely related to minimal surfaces are *surfaces of constant mean curvature*. The definition means what it says, with the one addition that the constant is assumed to be nonzero. Constant mean curvature surfaces minimize area subject to a volume constraint. The most famous example is the round sphere, which minimizes surface area subject to the constraint of enclosing a fixed volume. They appear as soap bubbles and, more generally, as interfaces when there is a pressure difference from one side to the other.

The Clay Summer School, of which I was the Director, was organized by a committee of researchers whose expertise spans the subject and its applications: Joel Hass, UC Davis; Arthur Jaffe (ex officio), Harvard and CMI; Rick Schoen, Stanford; Antonio Ros, Grenada; Harold Rosenberg, Paris VII; Mike Wolf, Rice. The first two weeks of the Summer School also functioned as MSRI's Summer Graduate Program on Minimal Surfaces, coordinated by Joel Hass. Over forty students from MSRI's sponsoring institutions were here for the first two weeks, together with about sixty Summer School participants, lecturers and mentors. It was an advanced-graduate introduction to the theory of minimal surfaces. The Clay student participants were chosen competitively from candidates nominated by senior mathematicians. Joining them were the lecturers, session assistants — all working researchers in the subject — and other senior mathematicians who attended the lecture series, served as mentors and gave special colloquia. (The last three weeks of the CMI Summer School, the "research component", will be discussed starting at the end of page 10.)

The featured event of the first two weeks of the meeting was Frank Morgan's two-week lecture course on *Geometric Measure Theory and the Proof of the Double Bubble Conjecture*. The conjecture says that the least-area way to enclose and separate two given volumes of air is the familiar standard double soap bubble shown on





the left, rather than some more exotic version such as the one on the right. Several years ago, Joel Hass and Roger Schlafly proved the conjecture when the two volumes are assumed to be the same. They used some important theoretical observations, including a key idea of Michael Hutchings, followed by a novel proof that employed interval programming. Last year, Hutchings, Morgan, Manuel Ritore and Antonio Ros announced a proof of the Double Bubble Conjecture for arbitrary volumes. Geometric measure theory was key to their work, as one is forced to consider surfaces with singular sets of an unpredictable nature.

Frank's lectures discussed modern, measure-theoretic definitions of surface, compactness of spaces of surfaces, and finally the proof of the Double Bubble Conjecture. Gary Lawlor ran problem sessions and Ritore was also in attendance, answering questions and preparing the written version of Morgan's lectures for the forthcoming proceedings of the Summer School.

Complementing Morgan's geometric measure theory were the innovative lectures of Matthias Weber, who spoke on the computational aspects of minimal surface theory. The course doubled as a high-level introduction to the classical representation of minimal surfaces using complex function theory and conformal mapping. The software he used was a combination of his own well developed Mathematica notebooks and a new Java version of MESH, a software suite developed by Jim Hoffman of MSRI. The computing staff at MSRI was extremely helpful in the deployment of this software for use in the course, doing all the necessary installation and system-wide adjustments to make it run on every office Linux machine. Weber's course and computing assignments were complemented by lectures by John McCuan who covered the basics of the subject. Special topics such as the conjugate surface construction, Schwarz reflection, associate families and periodic minimal surfaces were discussed in companion lectures by Hermann Karcher, Harold Rosenberg, Bill Meeks and myself.

There were numerous other lectures and tutorials during the first two weeks. MSRI's Bob Osserman gave a very good and very personal one: "A Historical Survey of Some Aspects of Minimal Surface Theory." Subtitled "One thing leads to another, or my 50 years with minimal surfaces," the lecture follows the development through Osserman's research of some of the central ideas of the subject in the last half of the Twentieth Century. Beginning with the question of the conformal type of an entire graph, connecting to the globalization of the Weierstrass representation and Osserman's characterization of complete minimal surfaces, it continues on to the Plateau Problem and beyond. You can enjoy this lecture yourself by going to http://www.msri.org/publications/ln/msri/2001/minimal/osserman/1/index.html.

In fact, all the lectures mentioned here, and the entire Summer School, can be seen on the Web, at http://www.msri.org/publications/video/index02.html. We expect to have the lectures available on CDs in the near future.

The Fourth of July fell during the second week of the meeting. After some lecture-demonstrations about the rules and practices of baseball for the participants who were visiting the USA for the first time, we went to see the Oakland A's beat the California Angels. On the bus to the Oakland Coliseum, all participants were coached then tested on the singing of "Take me out to the ball game." Consumption of Cracker Jacks and hot dogs was strongly encouraged. After the game, we all went to Jack London Square on the Oakland waterfront to roam around and to participate in the holiday festivities after which we enjoyed a banquet dinner at Le Cheval, a fine Vietnamese restaurant in Oakland's storied downtown. The participants made much of the fact that the restaurant is located on Clay Street, but I assure you that it was a complete coincidence. But I'm not sure what to make of the fact that during the seventh inning stretch, the scoreboard lit up welcoming, among others, the "CMI/MSRI Minimal Services" folks. We answered this challenge by singing "Take me out to the ball game" again.

All was not fun and baseball games that second week, however. Rick Schoen (a pretty good baseball player himself) gave a three-lecture course on the applications of mean curvature to general relativity and Riemannian geometry. The highlight of it for me was the discussion of the Penrose Inequality and description of the Huisken–Ilmanen and Bray method of solutions. (See box insert.) See also Schoen's lecture at http://www.msri.org/publications/ln/msri/2001/minimal/schoen/2/banner/01.html for the methods used to establish a proof.

Konrad Polthier (U. Berlin) gave three lectures about discrete minimal surfaces. The idea of a discrete minimal surface (Pinkall and Polthier) makes sense of what a harmonic map should be when the domain of definition is a polygonal surface. It has great value when one desires to approximate examples that are conjugate to minimal surfaces bounded by straight lines. There is now a quite impressive convergence theory for discretizations of surfaces.

The Penrose Inequality

The (Riemannian version of the) Penrose inequality goes something like this: Consider a metric on M^3, a three-manifold constructed by removing a finite number of balls from $\mathbb{R}^3$. The boundary of M^3 consists of the union of the boundary spheres of the balls. Suppose M^3 carries a metric of the form $g_{i,j} = (1+\frac{m}{2|x|})^4 + O(\frac{1}{|x|^2})$ (for $|x|$ large) that has nonnegative scalar curvature and the property that each boundary sphere is minimal. Assume in addition that M^3 contains no other compact minimal surface. Then $m \geq \sqrt{\frac{A}{16\pi}}$, where A is the sum of the areas of the boundary spheres. Equality holds only for the Schwarzschild metric: $g_{i,j} = (1+\frac{m}{2|x|})^4$.

Brian White lectured about three loosely related topics having to do with the existence and properties of solutions to the Plateau Problem. He presented in the second lecture a proof the following result (joint with Ekholm and Weinholtz): given a closed curve Γ in $\mathbb{R}^3$with total curvature equal to or less than 4π, then any minimal surface spanning Γ must be embedded. This result, whose proof is ingenious and conceptually elegant, implies or improves a number of classical results including the Fary–Milnor Theorem about unknottedness of curves of total curvature less than or equal to 4π, and Nitsche's Theorem stating that such a curve bounds a unique simply connected solution to the Plateau Problem. (See http://www.msri.org/publications/ln/msri/2001/minimal/schoen/2/banner/01.html.)

The most striking thing about the first two weeks of the CMI Summer School was the enormous amount of interaction between established mathematicians, graduate students and post docs. In addition to scheduled help sessions, voluntary group meetings were held after the day's lectures were done. Frequently, two or more groups of five to ten students met at the blackboards on the second and third floors with two or three senior mathematicians in attendance. These meetings often lasted until 6:30PM or later. Hermann Karcher had a series of meetings with a small group of graduate students that took place in various locations of MSRI. One of them, held on the MSRI patio, was so intense that the members were oblivious to call for participation in the group photograph, which had to be retaken after we found out that this fanatic study group had missed it.

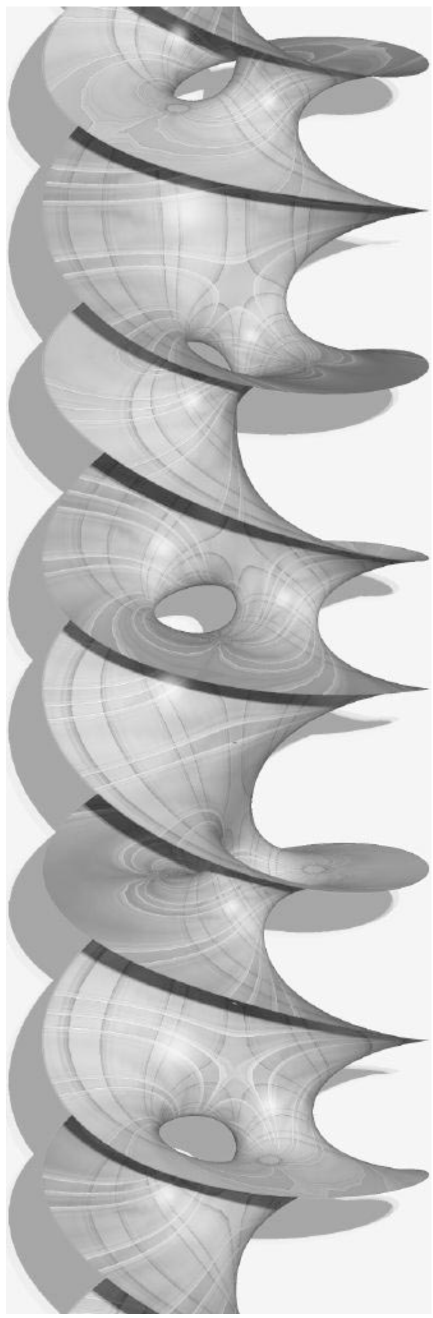
The response from students was extremely positive. Several students stayed longer than they had originally planned because of the excitement of the meeting and the intensity of the interaction with senior mathematicians. Here is one glowing comment, by Shabnam Beheshti, Department of Mathematics and Statistics, Texas Tech University:

"As I wrote on the check-out form, the best four weeks of my graduate life (thus far) just finished and I will be beginning my studies with a fresh perspective and renewed enthusiasm. I can only hope to bring a small piece of the wonderful excitement of the workshop back to my Department in the autumn."

The Research Component ran from July 9 to July 27. In the first week, Antonio Ros gave a series of lectures about the isoperimetric problem, a subject intimately tied the study of constant-mean curvature in Riemannian manifolds. The emphasis was on methods: symmetrization, suitable in spaces with many symmetries; stability, most useful in negatively curved spaces; and comparison with model situations, including novel comparisons to Gaussian measure models. Ritoré gave the last lecture of this series, on optimal inequalities in Hadamard–Cartan manifolds.

Joel Spruck and Bo Guan gave a series of lectures about the geometry of fully nonlinear elliptic equations, the prototype of which is the Monge-Ampere equation. Among the topics covered were the global study of hypersurfaces of constant, positive Gauss curvature and Weingarten hypersurfaces. During the entire series, there were many open problems mentioned.

Prominently discussed were new methods of constructing minimal surfaces, both from the point of view of PDEs and the perspective of the Weierstrass representation and complex function theory. Nicos Kapouleas and (separately) Rafe Mazzeo and Frank Pacard outlined their work on what could be termed gluing together embedded minimal surfaces to construct new examples, and desingularizing the intersection of embedded minimal surfaces of finite total curvature. Pascal Collin and Karsten Große-Brauckmann described various existence and structure results for constant mean curvature surface in Euclidean and hyperbolic space. Martin Traizet described an elegant theory of deformation families of embedded minimal surfaces of finite total curvature in flat three manifolds and their singular limit configurations. Within some natural restrictions, he showed that one could prove, by a process of opening nodes in the limit configuration, the existence near the limit of a deformation family. He used this to produce a higher genus analog of the classical Riemann minimal surface. Even more impressive, Traizet used this technique to produce a complete embedded minimal surface of finite total curvature in $\mathbb{R}^3$ with no nontrivial symmetries. All previous examples, ever since Euler discovered the catenoid in 1744, had been found by assuming symmetry in order to simplify the analysis. Traizet's announcement of this result was received with a round of applause.



Matthias Weber and Mike Wolf presented their theory of flat structures, which allows handle addition to minimal surfaces. Other methods from Teichmüller theory are used to develop a description of the moduli space of examples and to solve the period problem for minimal surfaces in a geometric fashion. A strong application of these new techniques is the proof of embeddedness of a genus one helicoid as the limit of a deformation family of screw-motion-invariant surfaces (such as the one shown here) with all the properties of the one shown to exist by Hoffman, Karcher and Wei (joint result of Weber, Wolf, and myself.) This surface is the first properly embedded minimal surface with infinite total curvature to be found since Meusnier showed that the helicoid was minimal in 1776. Matthias, who gave also gave a series of lectures during the first week of the Summer School, was a busy man. On July 2 (perfectly timed to take place between his two lecture series), his wife Silja gave birth to a beautiful baby girl they named Mirja.

Bill Minicozzi gave five lectures in which he developed his theory of embedded minimal surfaces of finite toplogy in a ball of $\mathbb{R}^3$. This represents joint research with Toby Colding. Primarily, he discussed the case when M is a disk. The key analysis concerns behavior of M near points where the curvature becomes large; a multi-graph forms near such points, very much like a piece of a helicoid near its axis. Minicozzi explained the different scales on which one sees this multigraph, and from this perspective there is emerging a global picture of the geometry of M. The main tools are the stability equation and the conformal geometry of minimal surfaces. This work leads to their theory of limit minimal laminations for blow-downs of uniformly locally simply conected minimal surfaces.

The Summer School showed that the study of minimal surfaces is in a period of dramatic progress. Example: Harold Rosenberg and Bill Meeks proved in their lectures that the helicoid is the only non-planar properly embedded and simply connected minimal surface. No assumption is made about the conformal type of the surface or the behavior of the Gauss normal map at infinity. The proof involves a marvelous combination of geometric and analytic techniques; a key step makes essential use of the new analytic results of Colding and Minicozzi.

The Meeks–Rosenberg proof shows that a properly embedded minimal surface of finite topology with one end is asymptotic to the helicoid. This naturally raises the question: *Does every such surface have a symmetry, as the helicoid does?* This seems like a long shot, although the evidence from related cases is mixed. One the one hand, if one replaces the assumption "with one end" by "with more than one end", the answer is no, as shown by Traizet's construction of a complete embedded minimal surface of finite total curvature with no symmetries. One the other hand, there are indications that it may be yes in the case of genus one discussed by Weber and Wolf.

The lectures of Joyce, Schoen, and Wolfson discussed the mathematics of lagrangian and special lagrangian geometry. The variational theory of the volume functional among lagrangian competitors was presented as an approach to constructing minimal lagrangian and special lagrangian cycles in Kähler–Einstein manifolds. The importance of special lagrangian cycles in string theory was discussed including a description of the SYZ geometric construction of a mirror for a Calabi–Yau threefold.

Joel Hass and Hyam Rubinstein lectured on the many applications of minimal surfaces to

(continued on page 19)

The Archimedes Society

David Eisenbud

Last year, MSRI received its fifth operating grant — now called a "Cooperative Agreement" — from the National Science Foundation. A new requirement for future funding is that MSRI take steps towards eventual financial independence. Though this is a serious burden, it seems to me a healthy direction: to best serve the mathematical community, MSRI needs a certain measure of independence. To move in this direction, we have hired Jim Sotiros as Development Director, and MSRI trustees have organized committees and programs to look at capital needs (including the building) and operating needs of the Institute.

The MSRI Archimedes Society is being inaugurated to diversify our funding, and to recognize donors to MSRI's Annual Fund in a wide range of levels. Donors of $34 to $89 will be recognized in the Fibonacci Level; $90 to $257 in the Fermat Level; $258 to $727 in the Cubic Level; $728 to $1,729 in the Ramanujan Level; $1,730 to $1,921 in the Noether level; $1,922 to $5,040 in the Plato Level; and $5,041 and above in the Museion level. As you can probably tell, mathematicians have already gone to work crafting our donor levels (see http://www.msri.org/development/membershiplevels.html for the explanation of these funny numbers.)

The financial support of MSRI's alumni — its former members and workshop participants — will mean much more than satisfaction of NSF stipulations. MSRI has substantial needs that cannot be met with government funding. Let me share some of them with you:

Afternoon Tea is a prime place for the sort of informal contact that makes MSRI special. It's important to MSRI's culture and to the interaction of our researchers. Although everyone, including NSF program officers, acknowledge this, we are forbidden to use NSF support for this central institution.

Our building, put up on a shoestring budget almost 15 years ago, needs increasing attention. To live up to MSRI's standing in the mathematical world, it also needs some renovation and improvement. The NSF will fund such improvements only in a very limited way.

Most important is MSRI's independence. The NSF has been a good patron over the nearly twenty years MSRI has operated, but having only a single funding source is not healthy. It can stifle creativity; and it can put us at the whim of priorities set by non-mathematicians.

There is a need for a fund for Director's Initiatives. Opportunities arise regularly — scholars who could be included in a workshop; an expenditure that will generate a ten-fold return; a researcher with a special need — that we simply have to let go, because there are few flexible funds available to achieve special goals.

Lastly, fund-raising success breeds success: many foundations we will be approaching ask for information on the percentage of members who participate in Annual Fund drives. Some will only give when a large number of an Institute's "family" have already decided to give. So, a gift from you to the Archimedes Society — at any level — could help make possible many other gifts to accomplish MSRI's objectives.

The Archimedes Society will help ensure a secure and promising future for MSRI. We respectfully ask that you include MSRI this year in your budget for philanthropy! Join the Archimedes Society today by responding with the envelope included with Emissary, look for a mailing from us in December, or go to www.msri.org/development to learn more and make your gift via secure server.

I thank you, in advance, for your assistance and support.

Mathematics in the Media

Sara Robinson

The portrayal of mathematics in the media is a subject of great concern to the mathematical community, judging from the heated discussions at a math and the media workshop and panel convened at the SIAM annual meeting in July.

Mathematicians said they are frustrated by the media's priorities in science coverage, and the short shrift mathematics gets in comparison with physics and biology. Journalists countered that mathematicians aren't doing enough to make their contributions sound interesting and relevant to the general public.

The workshop, sponsored by SIAM and MSRI, primarily consisted of a panel discussion with reporters and editors from the New York Times, The Wall Street Journal, and Interactive Week, a media relations specialist from IBM Almaden, and a mathematician: Joe Buhler, then Deputy Director of MSRI.

The first hint of controversy came early when the reporters and editors explained their criteria for what makes a good story on a scientific or mathematical topic.

Don Clark of the Wall Street Journal said his main criterion is whether the result could lead to substantial improvements in an industrial process, measured in dollars.

Rob Fixmer, editor-in-chief of Interactive Week and a former editor at the New York Times said he thought it was most important to cover topics connected to large expenditures of taxpayer dollars, such as NASA's space exploration programs.

Offering a ray of hope to the pure mathematicians present, John Markoff of the New York Times, said he likes to write about compelling personal stories associated with research. The example he gave is the struggle between Edward Felten, a computer science professor of Princeton, and the recording industry over the publication of his encryption research. Mathematicians are probably more familiar with the media attention given to the tale of Andrew Wiles, working on a 400-year-old math problem in his attic.

From the audience of mathematicians came an outpour of dismay. One mathematician said he was disillusioned to hear that reporters and editors of even the most prestigious publications are driven by such ignoble concerns. The elite newspapers have a responsibility to educate people about the world they live in, he said.

At this point, Joe Buhler chimed in. Mathematicians need to "get over it," he said, since, like it or not, the media doesn't see promoting mathematics awareness as one of its responsibilities. Reporters will write about things that they and their readers find interesting and intelligible, he added, and it's up to mathematicians to put their research in that category.

Many members of the audience expressed some degree of physics-envy, noting that "gee-whiz" articles on results in cosmology or particle physics are common, even though those fields also have no direct applications. Others noted that while mathematics is behind many seminal results in other fields, such as biology, engineering, and physics, somehow all the credit goes to the applied sciences, not the underlying mathematics.

From there, the discussion moved to accuracy in the media after Buhler provided some welcome comic relief with his story about being interviewed for the New York Times for an article on juggling. Buhler said he told the reporter that mathematicians and "people who like algorithms" tend to be juggling fans, but when the article was published, his phrase came out as "people who like logarithms."

Initially, Buhler said, he was mortified to see himself quoted in a major newspaper in a way that made him look silly. But Claude Shannon, also interviewed for the article, pointed out that while this sort of mistake makes mathematicians cringe, for most of the readers of the New York Times the words "algorithm" and "logarithm" are more or less equivalent. Overall, Shannon said, the article was a success.

Here, Mike Ross, of IBM Almaden, stepped in and described what he does to increase the chances that IBM researchers will be quoted correctly in interviews. In pre-interview training sessions, he has the researchers select a few main points they want to get across and figure out a clear, yet non-technical way of summarizing them.

The tendency with researchers, Ross said, is to tell the story of the result chronologically without making the main point clear. By starting with the main point, already translated into language for the layperson, Mr. Ross said, even if the reporter doesn't understand very much about the work, they will at least come away with something accurate.

The workshop concluded with an exercise: Members of the audience were asked to write story ideas down on a piece of paper and the panelists looked through the ideas and said which ideas were more or less appealing and why. The media panelists then chose the idea they found most compelling, one on the mathematics of gravitational waves, and conducted a short, sample interview with its author: Douglas Arnold, the incoming director of the Institute for Mathematics and its Applications in Minneapolis.

Arnold's idea was a story on the difficulty in devising algorithms to find solutions to the equations linking gravitational waves to cosmological events. In his description of the story idea he played up the physics of gravitational waves, generating some complaints from the audience that the resulting story would be about the physics, rather than the underlying mathematics.

Other audience members pointed out that it would be tough to describe the mathematics in an article for the general public.

At this point, the workshop ended, but the workshop moderator, me, had some further discussions with the journalists about their impressions from the workshop.

Fixmer, of Interactive Week, said he hadn't realized mathematicians feel snubbed by the press, compared with the other sciences.

"My first impression was that these mathematicians are awfully defensive – they kept comparing themselves to physicists and wondering why everyone loves physicists and no one loves them," he said. "What physicists do challenges people's notions of origins and creations. Math doesn't challenge any fundamental beliefs or what it means to be human."

When I told him that mathematics does all these things, Fixmer said that still, people don't perceive math as being in any way connected to them or the things they care about. His recommendation to the mathematics community is to take a proactive role in making journalists aware of its contributions.

He suggested that mathematics organizations should send out press releases every time there's an announcement or discovery in science or technology where math has played an important role.

"Physicists really have it good," Fixmer said. "I didn't realize it before the conference, but they play an amazing, easily recognizable role. No one understands how math fits in with other disciplines so mathematicians need to do a better job of explaining it."

He Explains the Book Proof

Sandra M. Gilbert

The shadowy clatter of the café
frames the glittering doorway.

A white cup and a blue bowl
inscribe pure shapes on the table.

The mathematician says, Let's turn the pages
and find the proof in the book of proofs.

He says, It's as if it's already there,
somewhere just outside the door,

as if by sitting zazen in a coffeehouse,
someone could get through or get "across,"

or as if the theorems had already all
been written down on sheer

sheets of the invisible,
and held quite still,

so that to think hard enough
is simply to read and to recall —

the way the table remembers the tree,
the bowl remembers the kiln.

This poem and the ones on pages 5 and 7 are taken from poet and literary critic Sandra Gilbert's *Kissing the Bread: New and Selected Poems, 1969–1999* (Norton, 2000). They appear in the section titled "When She Was Kissed by the Mathematician".

A New Generation of Signal Processing and Signal Processors

D. M. Healy Jr. and D. N. Rockmore

In many ways, the late 1950's marked the beginning of the digital age, and with it, the beginning of a new age for the mathematics of signal processing. High speed analog-to-digital converters had just been invented. These devices were capable of taking analog signals like time series (think of continuous functions of time like seismograms which measure the seismic activity — the amount of bouncing — at a fixed location, or an EEG, or an EKG) and converting them to lists of numbers. These numbers were obtained by *sampling* the time series, that is recording the value of the function at regular intervals, which at that time could be as fast as $300,000$ times every second. (Current technology permits sampling at much higher rates where necessary). Suddenly, reams and reams of data were being generated and new mathematics was needed for their analysis, manipulation and management.

So was born the discipline of Digital Signal Processing (DSP), and it is no exaggeration to say that the world has never been the same. In the mathematical sciences the DSP revolution has, among other things, helped drive the development of disciplines like algorithmic analysis (which was the impetus behind the creation of computer science departments), communication and information theory, linear algebra, computational statistics, combinatorics, and discrete mathematics. DSP tools have changed the face of the arts (electroacoustic music and image processing), healthcare (medical imaging and computed imaging), and of course, both social and economic commerce (i.e., the internet). Suffice to say that the mathematics of DSP is one of the pillars supporting the amazing technological revolution that we are experiencing today.

Now, as we turn the corner on the twenty-first century, some fifty years after the birth of the digital age, we find ourselves facing an analogous paradigm shift in the world of signal processing. With the goal of introducing a new generation of mathematicians to the exciting new frontiers of signal processing, we ran a Summer Graduate Program in Modern Signal Processing at MSRI during the first two weeks of June.

The growing mathematical sophistication of the signal processor's toolbox is driven by the need to collect, manage, analyze and exploit data which is growing in complexity, size, and diversity. This data is presented in a variety of formats ranging from simple one-dimensional time domain signals, through the 2-D, 3-D and 4-D realms of basic imagery and video, and on to increasingly higher dimensional data of various sorts. Analytic tools and representation techniques whether they be new or old, are coupled with powerful ideas from statistics, optimization, geometry, and other mathematical disciplines in order to provide us with powerful and systematic approaches to the problem of detection, enhancement, and exploitation of useful information hidden in data. The challenges that increasing dimensionality presents for the efficient processing and generation of meaningful inference is a critical concern in modern signal processing and one that was a focus for the MSRI graduate program we presented.

Until fairly recently, many DSP insights and tools drew heavily on the methods and perspectives of classical Fourier analysis: the decomposition of any function (eg., time series, sound, radar return) into a sum of sines and cosines of varying frequencies and amplitudes. This sort of spectral analysis was useful for many of the original security-driven DSP applications such as the ability to detect nuclear test blasts in the interior of the Soviet Union or finding Soviet submarines via sonar. The development of the Fast Fourier Transform (FFT) made possible the highly efficient transformation of sample data into spectral data. The extreme computational advantages of these algorithms and relative success of the spectral approach in many problem domains have led researchers and engineers to rely on these sorts of spectral methods.

Beyond these particular computational tools, Fourier and related spectral analysis ideas also helped form the intellectual paradigm guiding development of other DSP methodologies. For example, Weiner's work on generalized harmonic analysis provided important impetus to some early statistical signal processing ideas. Many of the time-domain and state-space tools of signal processing were analyzed using perspectives from Fourier and related spectral analysis ideas.

Ubiquitous networked computing has once again changed the order of magnitude of the size of datasets that are routinely interrogated and manipulated by an increasingly large and diverse set of customers. New sensing modalities are changing the form of the data that researchers need to consider, as well as making necessary techniques for integrating data from different sources in order to best understand the information they contain. While spectral methods are still of vital importance for many applications, it has also become increasingly clear that pure sinusoidal representations may not provide the best description of certain classes of functions. The analysis of many kinds of digital data is now often augmented by more general time-frequency filterbank ideas such as adaptive wavelet-based methods. In some cases, these techniques can provide much more simple and computationally efficient representations than those obtained via classical Fourier-based methods. Particular examples include the success of wavelet-based techniques for image compression and processing. Lastly, completely new application areas, such as molecular biology and quantum information science, are presenting disruptive challenges to current concepts of signal processing and control.

At a coarse scale, the Graduate Summer Program was divided into two components: fundamentals and current research. Mornings were devoted to introductory lectures on some of the standard mathematical tools which currently comprise DSP. This included a brief tour of Fourier-based DSP (sampling theory, algorithms), group theoretic generalizations and several lectures on adaptive wavelet-based approaches. The basics in the analytic methods of signal representation were coupled with some elementary lectures on statistical signal processing ideas including detection and estimation theory and information theoretic tools including source and channel coding (i.e., data compression and error-correction).

Morning lectures were supplemented with computational assignments from the Matlab software package.

Afternoons were given over to a host of outside speakers representative of the wide range of disciplines and methodologies that currently comprise the frontier of signal processing, as well as the wide range of working environments and application areas where signal processing is studied (academia, industry, defense laboratories, etc.).

For example, the field of Cognitive Neuroscience now uses many kinds of computed imaging techniques to accomplish non-invasive studies of brain activity. M. D'Esposito (UC Berkeley), the Director of UC Berkeley's new neuroimaging laboratory gave an overview of current mathematical challenges in the processing of fMRI data. P. Kostelec (Dartmouth and the National fMRI Data Center) explained related image processing problems in image registration, which is the problem of aligning similar images to a fixed template. These problems are also relevant for many defense applications and L. Peterson and G. Hewitt (USN Surface Warfare Center, China Lake) explained the use of level-set methods for registration and the related problem of motion detection and estimation.

Computed imaging has a long history in the field of medical imaging, and as technology develops, new kinds of data are generated and require new mathematical tools for their analysis. A. Grünbaum (UC Berkeley) gave an introduction to classical computed tomography (better known as CT) and the new questions raised by the new field of diffusion tomography in which tissue density data is acquired by mathematical analysis of the scattering of relatively low energy radiation.

Researchers are now taking advantage of relatively new analytic and computational tools which are well-adapted to particular signals and signal classes of interest. We are no longer in a one size (i.e., Fourier analysis) fits all paradigm. N. Saito (UC Davis) explained some of the techniques for choosing among different types of signal representations so as to optimize the sparseness or statistical independence of the resulting description. D. Donoho (Stanford) discussed new mathematical tools for Lie group-valued data (eg., orientation data) and the analysis of networks. These methods offer an exciting generalization of the wavelet-motivated multiresolution framework.

The enormous volume of data now transmitted and generated in various settings mandate the development of new tools for data compression and management. J. Li (Microsoft Research) gave a detailed overview of the mathematical tools that underlie the new JPEG 2000 image compression standard including wavelet descriptions of the data and the attendant quantization schemes. S. Evans (UC Berkeley) explained a novel application of Fourier analysis which made possible the reduction and analysis of a seemingly computationally intractable biology problem in the study of phylogeny trees.

Lastly, new sensing and communications media require new mathematical tools. For example, an understanding of the non-linear effects which can degrade optical signaling is becoming a critical issue in optical fiber communication. These issues were discussed by U. Österberg (Dartmouth College) who spoke on the challenges of signal design relevant to signal transmission over fiber optics. The exciting new computational framework of quantum computing was the subject of two introductory lectures by D. Meyer (UCSD). Quantum information theory bears some similarity to the classical Shannon theory but seeks to exploit the non-classical effects found in physical information systems at the quantum mechanical scale. Meyer described the quantum Fourier transform and a possible application to quantum error correcting codes.

System designers are beginning to think more carefully about optimizing the division of labor between signal sensing and processing. C. Priebe (Johns Hopkins University) gave introductory lectures on new approaches to the problem of feature classification relevant to this new field of "Integrated Sensing and Processing" (ISP). Many biological systems provide beautiful examples of "natural" ISP and the analysis and exploitation of biologically-based circuitry is one of the new exciting frontiers of signal processing. Lectures on the amazing possibilities and mathematical challenges related to cellular signaling and biochemical regulatory networks were delivered by A. Arkin (UC Berkeley and Lawrence Berkeley Laboratory).

The wide range of topics presented at the summer program reminded us of the famous 1968 Arden Conference devoted to the FFT. This meeting brought together mathematicians and statisticians, physicists interested in astronomical and atomic calculations, M.D.s interested in spectral analysis for medical sensing, radar and sonar engineers, hardware and software developers, and exemplified the excitement that surrounded the birth of the digital revolution. In a June 1967 special issue of the the IEEE Transactions on Audio and Electroacoustics (which has turned into the IEEE Transactions on Signal Processing), B. Bogert closes his guest editorial with the words "What lies over the horizon in digital processing is anyone's guess, but I think it will surprise us all". As the summer program showed, we now find ourselves peering towards a new horizon of signal processing, with a renewed sense of wonder, mystery and excitement.

Featured in This Issue

Zvezdelina Stankova, a postdoc at MSRI in 1997–98 and now an Assistant Professor at Mills College, continues her involvement with MSRI via the Berkeley Math Circle (see page 4).

Recent Public Events

Music

January 29, 2001: Music of Beethoven, Brahms and Schumann, performed by David Eisenbud, Louis Rowen, and Martha Wasley.

April 13, 2001: Award-winning Belgian pianist Jeanne Stark-Iochmans played Debussy's Etudes.

Art

July 13 to August 23, 2001: The Drawings of Guillaine Querrien.

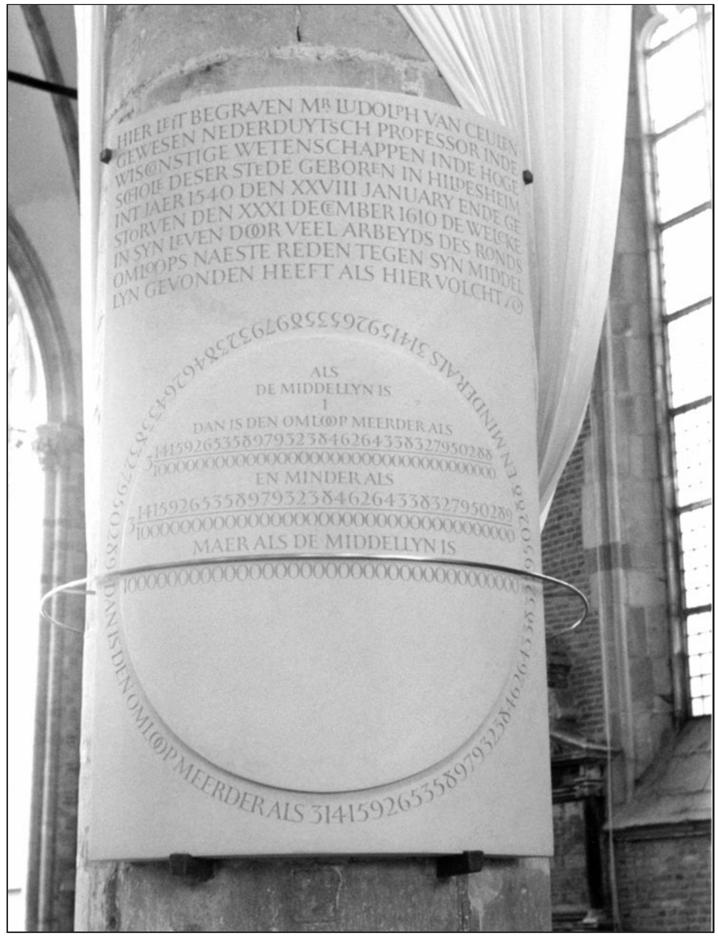
July 16, 2001: *Fermat's Last Tango*, West Coast premiere of the filmed Broadway play (see below for details).

Conversations

May 9, 2001: Conversation between Mathematics Teachers and Researchers: A discussion of the school program *Making Mathematics*.

Public Lectures

April 1, 2001: *Pi in the Pieterskerk*, by Dr. Hendrik W. Lenstra, Jr. (University of California, Berkeley, and Hewlett-Packard Visiting Research Professor at MSRI). As mentioned by the lecturer, on July 5, 2000 the Dutch Crown Prince unveiled in the historic Pieterskerk in Leiden a stone commemorating the Dutch mathematician Ludolph van Ceulen (1540–1610). Van Ceulen achieved fame for his accurate computation of the number π, the ratio between the circumference and the diameter of a circle. Remarkably, his result was first made public on his tombstone, a unique mathematical monument that was lost in the early nineteenth century. Prof. Lenstra described the rediscovery of the original text on the tombstone and his initiative to have a replacement copy made and installed in the Pieterskerk.



From the inscription:

"Here lies Mr. Ludolph van Ceulen … When the diameter is 100000000000000000000000000000000000, the circumference is greater than 314159265358979323846264338327950288 and less than 314159265358979323846264338327950289."

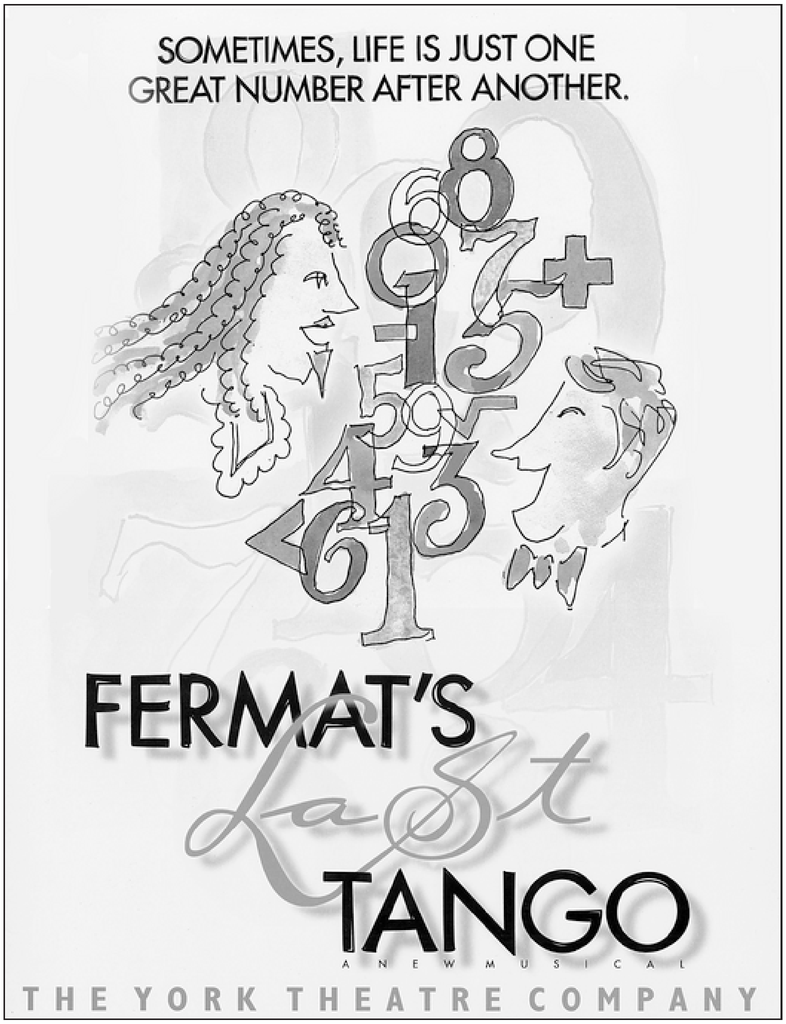
First *Last Tango* in Berkeley

David Hoffman

On July 16, 2001, the screen version of *Fermat's Last Tango* had its world premiere before an audience of about 500 in the Roda Theatre, Berkeley Rep's dazzling new playhouse. *Fermat's Last Tango*, a musical drama based on the recent proof of Fermat's Last Theorem by Andrew Wiles (music by Joshua Rosenblum, book by Joanne Sydney Lessner, lyrics by Lessner and Rosenblum), was produced by the York Theatre Company and ran Off Broadway in late 2000 and early 2001. Under the direction of Arthur Jaffe, the Clay Mathematics Institute taped a live performance, from which they produced a professionally edited version of the show in DVD format. (The DVD also contains a beautifully produced and edited interview with Wiles, and is packaged with an informative booklet.) The premiere screening used a master DVD, a 20×24 rear-projection screen and the Roda's state-of-the-art sound system. The quality was comparable to that of a first-run movie theatre.

I can hear your questions now as I write this article, weeks or months before you read it in the Emissary: A musical about Wiles' proof of Fermat's Last Theorem on DVD? What? Are you serious? True, it's hard to imagine what such a production could be like. I certainly had no idea what to expect the first time I saw it. So, before I tell you more about the evening, let me tell you something about the musical.

In *Fermat's Last Tango*, Daniel Keane (Wiles) earns overnight acclaim when, after many years of solitary labor in his attic, he announces his proof of Fermat's Last Theorem. Elation soon gives way to doubt when a problem arises in the proof. In the musical, Fermat — who is portrayed as rather mean-spirited — drops in on Keane soon after the announcement. Fermat is not happy about this turn of events. He takes Keane on an unsettling journey to the *Aftermath*, inhabited by Pythagoras, Euclid, Newton and Gauss. (Try to imagine the apotheosis of a Ph.D. oral examination staged by the Marx Brothers.) There Keane is taunted, tested, then told that his proof contains a gap — "a big fat hole", sings Fermat.



Consumed by the obsession to fill in the gap, Keane — in crisis — retreats again to his attic. He is haunted by Fermat, who has a personal interest in his failure, and challenged by his wife, who, while supportive, wants him to appreciate her curves, too. Her number, *Math Widow*, produced cries of recognition from a few spouses in the audience seated next to some of the many mathematicians present. The three of them perform the title tango in what may be the play's most moving scene.

Except for Keane, the Wiles character, all mathematicians involved are mentioned by name, including Ken Ribet who was on hand for the Q&A session after the screening. With one or two exceptions, the authors get the history of the subject fairly straight (and, yes, there are lines in which *Taniyama–Shimura* is sung). Of course, the *Aftermath*, the relationship between Keane and his wife and the character of Fermat are fictional creations and there a some campy moments, but something of the spirit of mathematical struggle does come through.

The occasion for the screening was the Clay Mathematics Institute Summer School on the Global Theory of Minimal Surfaces, of which I was the Director (and about which you can read more on page 1). This was the Summer School's public event, and it was jointly presented by CMI and MSRI. Seeing *Fermat's Last Tango* in a theatre with a large audience, many of whom were mathematicians, was an unusual experience. The evening began with a few minutes of minimal-surface animations to music, after which Arthur Jaffe, President of CMI, provided some background for the achievement of Wiles and the creation of this *musical fantasy*. Then the interview with Wiles (in Paris on the occasion of the CMI Millennium Event in 2000) was shown. It made concrete for the general public the nature of mathematics research and the atmosphere in which it is done. By introducing Wiles himself to the audience, it also served as a counterpoint to the sometimes-frenetic dramatic conflict that followed in the musical. In the interview, Wiles is poised and lucid, magisterial in a quiet, calming way. The masterful editing of the interview and the setting (a vaulted room in the College de France) reinforced this impression.

After the screening, I moderated a 45-minute question-and-answer session with Ken Ribet of UC Berkeley (whose work, showing that Fermat's Last Theorem was a consequence of the Taniyama–Shimura Conjecture, is mentioned in the play), Karl Rubin of Stanford (an expert algebraic number theorist and Wiles' first doctoral student) and Arthur Jaffe. Most of the questions were from nonmathematicians: it was quite clear that they were captivated by this presentation of mathematics as a human activity. After the show, I asked a few mathematicians to tell me their impressions of *Tango*. One was struck by the portrayal of mathematical obsession that is so often part of mathematical research, and felt that it came through, in a moving way. Another was glad that his wife was not there to hear *Math Widow* (she was home, 2000 miles away while he was at the CMI Summer School).

Wiles, in the interview mentioned above, said:

> ... We feel a real part of history. Mathematics has been going on for thousands of years and we still use results that were proved a thousand or two thousand years ago. And so we feel part of an enterprise that's really been going on for a long time and will go on, we believe, forever. There are very few other enterprises where you feel this permanence. Countries come and go, political systems come and go, but somehow mathematics has a permanence that almost nothing else has.

Certainly, Wiles was thinking about Greek mathematics. There is another enduring human activity whose tradition is the Western World begins in Greece: theatre. That night we saw these two traditions come together briefly, in an attempt to understand the human component and cultural meaning of a great intellectual acheivement. It was a special evening.

The Clay Mathematics Institute will offer a VHS tape as well as a (higher quality) DVD of Fermat's Last Tango to the general public at cost. This package will include a video recording of the play, a video of an interview with Andrew Wiles in May 2000, and a pamphlet both about the production and about the history of Fermat's Last Theorem. See http://www.claymath.org/events/fermatslasttango.htm for the latest information about availability.

Current and Forthcoming Workshops

August 13 to 24, 2001: *Introductory Workshop in Inverse Problems and Integral Geometry*, organized by Liliana Borcea, David Colton, Michael Eastwood, Simon Gindikin, Alexander Goncharov, and Gunther Uhlmann.

October 8 to 12, 2001: *Integral Geometry in Representation Theory*, organized by Leticia Barchini and Roger Zierau.

October 29 to November 2, 2001: *Pan-American Advanced Studies Institute (PASI) on Inverse Problems*, organized by Gunther Uhlmann (chair), David Haynor, Gary Margrave, and Ricardo Weder.

November 5 to 16, 2001: *Inverse problems and Applications*, organized by Joyce McLaughlin, Adrian Nachman, William Symes, Gunther Uhlmann (chair), and Michael Vogelius.

November 27 to December 1, 2001 at The Banff Centre in Alberta, Canada: *Automorphic Forms and Representations of p-adic groups*, organized by W. T. Gan, J. S. Li, D. Ramakrishnan, G. Savin (chair), and J. K. Yu.

December 10 to 14, 2001: *Special values of Rankin L-series*, organized by Henri Darmon and Shouwu Zhang.

January 10 to 16, 2002: *Introductory Workshop on Algebraic Stacks, Intersection Theory, and Non-Abelian Hodge Theory*, organized by William Fulton, Ludmil Katzarkov, and Tony Pantev.

February 25 to March 1, 2002: *Information Theory*, organized by Elwyn Berlekamp, Joe Buhler, Dave Forney, Gadiel Seroussi, Andy Viterbi, Sergio Verdu (chair), and Marcelo Weinberger.

March 11 to 15, 2002: *Intersection Theory on Stacks*, organized by K. Behrend, W. Fulton, L. Katzarkov, M. Kontsevich, Y. Manin, R. Pandharipande, T. Pantev, B. Toen, and A. Vistoli.

March 18 to 22, 2002: *Geometric Aspects of the Langlands Program*, organized by E. Frenkel, V. Ginzburg, G. Laumon, and K. Vilonen.

April 1 to 5, 2002: *Non-Abelian Hodge Theory*, organized by M. Kapranov, L. Katzarkov, M. Kontsevich, D. Orlov, T. Pantev, C. Simpson, and B. Toen.

April 15 to 26, 2002: *Conformal Field Theory and Supersymmetry*, organized by G. Felder, D. Freed, E. Frenkel, V. Kac, T. Miwa, I. Penkov, V. Serganova, I. Singer and G. Zuckerman.

May 6 to 10, 2002: *Meta-Analysis of Medical and Health-Policy Data*, organized by Joseph C. Cappelleri, Joseph Lau, Ingram Olkin (chair), Diana Pettiti, Drummond Rennie, and Donna Stroup.

June 3 to 7, 2002: *Finsler Geometry*, organized by David Bao, Robert Bryant, S. S. Chern, and Zhongmin Shen.

September 9 to September 13, 2002: *Introductory Workshop in Commutative Algebra*, organized by Luchezar Avramov, Mark Green, Craig Huneke, Karen E. Smith and Bernd Sturmfels.

Spring and Summer 2001 Workshops

January 22 to 27, 2001: *Free Probability and Non-commutative Banach Spaces*, organized by P. Biane, D. Shlyakhtenko, R. Speicher, D. Voiculescu, E. Effros, E. Kirchberg, V. Paulsen, G. Pisier, Z-J. Ruan, and A. Sinclair.

February 9 to 11, 2001: *The Preparation of Math Majors in the First Two Years: A Curriculum Policy Workshop*, organized by David Bressoud, Steve Krantz, Jim Lewis, William G. McMallum (chair), and William Velez.

March 1 to 3, 2001: *Analysis Models and Methods: A Conference on Applied Mathematics and Computational Methods in Memory of Fred Howes*, organized by A. Chorin, I. Singer, and M. Wright.

March 12 to 16, 2001: *Geometric Aspects of Spectral Theory*, organized by Jean-Michel Bismut, Tom Branson, S.-Y. Alice Chang, and Kate Okikiolu.

March 19 to 29, 2001: *Nonlinear Estimation and Classification*, organized by David Denison, Mark Hansen, Chris Holmes, Robert Kohn, Bani Mallick, Martin Tanner, and Bin Yu.

March 30 to April 1, 2001: *Conference on Randomized Algorithms in Finance*, organized by Phelim Boyle, Mark Broadie, Joe Buhler, Russell Caflisch, Sanjiv Das, David Eisenbud, Philippe Jorion, Mark Rubinstein, and Domingo Tavella.

April 23 to 27, 2001: *Quantization and Non-commutative Geometry*, organized by A. Connes, J. Cuntz, N. Higson, G. G. Kasparov, N. P. Landsman, H. Moscovici (co-chair), M. A. Rieffel (co-chair), G. Skandalis, A. Weinstein, M. Wodzicki, and S. L. Woronowicz.

April 26 to May 1, 2001: *29th Canadian Symposium on Operator Algebras*, organized by Man-Duen Choi, Edward G. Effros, George A. Elliott (co-chair), Vaughan F. R. Jones, Henri Moscovici, Ian F. Putnam (co-chair), Marc A. Rieffel, and Dan-Virgil Voiculescu.

April 30 to May 4, 2001: *Critical Percolation and Conformally Invariant Processes*, organized by Oded Schramm and Yuval Peres.

May 7 to 11, 2001: *Geometric Scattering Theory and Elliptic Theory on Noncompact and Singular Spaces*, organized by Tanya Christiansen, Charles Epstein, Rafe Mazzeo, and Richard Melrose.

May 29 to June 1, 2001: *The Continuum Hypothesis*, organized by Hugh Woodin and John Steel.

June 4 to 15, 2001: *Modern Signal Processing (Summer Graduate Program I)*, organized by Dan Rockmore and Dennis Healy.

June 25 to July 27, 2001: *Clay Mathematics Institute Summer School on the Global Theory of Minimal Surfaces*, organized by Joel Hass, David Hoffman, Arthur Jaffe, Antonio Ros, Harold Rosenberg, Richard Schoen, and Michael Wolf.

June 25 to July 6, 2001: *The Global Theory of Minimal Surfaces (Summer Graduate Program II)*, organized by Joel Hass and David Hoffman.

Most of these workshops are offered under the auspices of one of the current programs (page 8). For more information about the programs and workshops, see http://www.msri.org/calendar.

Clay Summer School on Minimal Surfaces

(continued from page 11)

the study of three-manifolds. After Hass gave a survey of some of the spectacular achievements in this area over the past 25 years, Rubinstein discussed some recent investigations into Heegaard surfaces and hyperbolic manifolds using the study of minimal surfaces in these manifolds. Hass concluded with a lecture on exciting recent connections to the theory of computational complexity achieved in joint work with Ian Agol, Jeff Lagarias, and Bill Thurston.

Techniques developed in the study of minimal surfaces turn out to be useful in the search for minimal spanning trees, known as the Steiner tree problem. This has interesting applications in the optimal layout of mine shafts, an application discussed in two additional lectures by Rubinstein.

The three weeks of intense mathematics were not without refreshing respites. On the first evening of the Research Component, we had a banquet dinner at a trendy new restaurant in downtown Berkeley called, appropriately enough, *Downtown*. There was a buffet dinner served at MSRI on Friday, July 13th. That day was the opening of an exhibit of drawings by Guillaine Querrien, a Parisian artist. Her work hung on the walls of the Institute through mid-August. (See http://zeta.msri.org/calendar/specialevents/SpecialEventInfo/69/show_specialevent.) On July 16, the Summer School participants were part of the audience of the world premiere of *Fermat's Last Tango* (see page 16). After one day's lectures, the monologist Josh Kornbluth, came to MSRI to give a private performance of his one-man play *The Calculus of Change*, a hilarious and poignant account of his unsuccessful freshman year at Princeton as a math major.

On July 22, we boarded buses in the morning for an all-day trip to Monterey Bay, which included a stop for lunch and a swim in Carmel, a walking tour of Point Lobos State Preserve, and dinner in the Monterey Bay Aquarium, which we had all to ourselves. Our dining room was in a room one of whose walls (perhaps 80 by 30 feet) was the transparent side of an enormous tank filled with the denizens of Monterey Bay, including turtles, tuna, and many large sharks. We ate chicken.

The organizers and participants are grateful to the Clay Mathematics Institute for funding this Summer School, and to MSRI and its staff for hosting it. It was a memorable meeting that will have a lasting and positive effect on the subject.

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MSRI Governance

In August 2001, MSRI welcomed Michael F. Singer as its new Deputy Director. He will be serving as Deputy Director until August 2002 and as Acting Director from August 2002 to August 2003, while David Eisenbud is on sabbatical. We thank Joe Buhler for his wonderful help and contributions to MSRI as a Deputy Director from August 1999 to July 2001. MSRI also welcomed the following people to its governing bodies in the Spring of 2001:

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