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Symplectic and Contact Geometry Return to MSRI

Yakov Eliashberg and Eleny Ionel

Twenty-one years ago, in 1988/89, MSRI held its first program in symplectic geometry. It was an exciting time, just a few years after revolutionary discoveries of Conley–Zehnder, Gromov, Floer and others had led to creation of the whole new area of mathematics: symplectic topology.

The 1988/89 MSRI program was instrumental in helping formulate the main problems in and directions of development of the new field. The years since that program have witnessed great successes in symplectic and contact geometry and topology.

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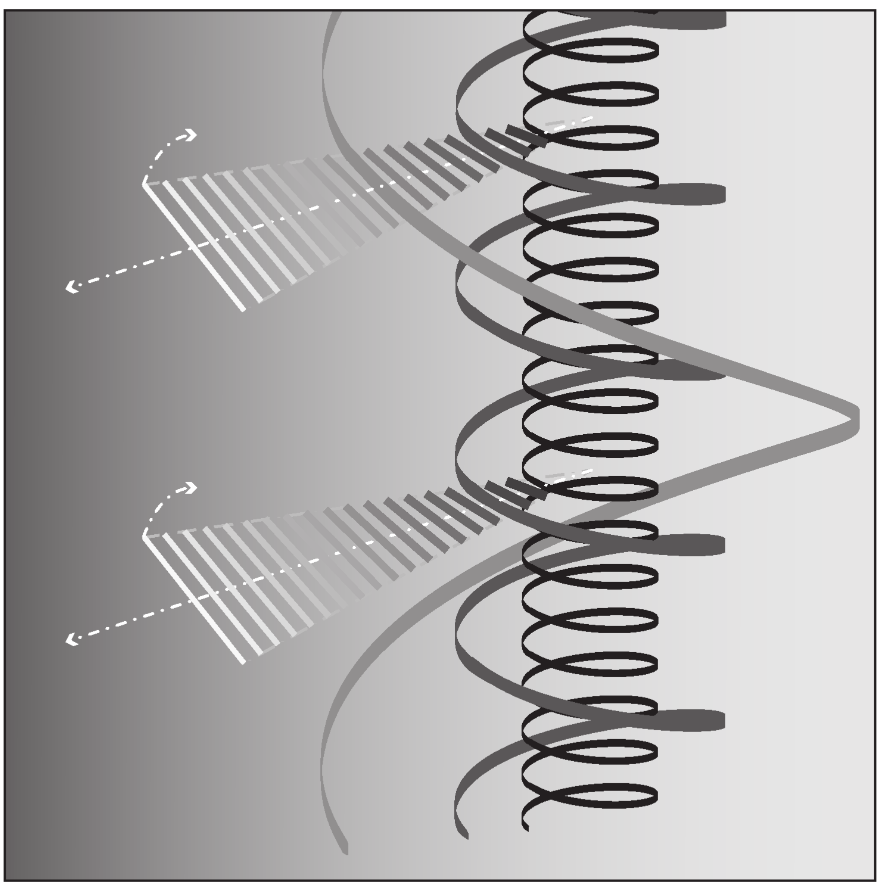

Geodesics in the subriemannian metric associated with the standard contact structure on $\mathbb{R}^3$. Art by Gregoire Vion, courtesy of Richard Montgomery.

Tropical Geometry

Ethan Cotterill

Tropical geometry is a new branch of mathematics, with roots in algebraic geometry and geometric combinatorics. It is closely related to, but distinct from, analytic geometry (of Berkovich), the geometry of formal schemes (of Grothendieck), and geometry over $\mathbb{F}_1$, the "field with one element" (of Manin). It has already proven to be a very useful tool in understanding the variation of algebraic varieties in families. Participants in the MSRI program in tropical geometry also witnessed the subject developing in a number of other significant directions, including representation theory and the study of Teichmüller space. The subject is enjoying a surge of activity and seems poised to remain of intense interest for the foreseeable future.

One of the most appealing aspects of tropical varieties is that their global properties are manifestly visual. The same cannot be said of algebraic schemes! The geometric amenability of tropical varieties, in turn, comes from the fact that they are polyhedral complexes. Formally, they may be constructed in at least two different ways, each of which has its merits. One route is via geometric dequantization and involves degenerating the base field of interest via a 1-parametric family of logarithms. This approach was first developed by Viro in his pioneering work on patchworking hypersurfaces over the real and complex numbers.

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Contents

Women and Mathematics

Why are so few top-level math research professors women, and what do we do about it?

Julie Rehmeyer

The mathematics faculty at top research universities is overwhelmingly male, despite decades of efforts to bring more women into math. Why, and what can be done about it?

At MSRI's April conference on math circles (see page 7), two scientists presented research to shed light on those questions. Janet Mertz, of the University of Wisconsin-Madison, presented evidence that there are many women with profound aptitude in mathematics and that environmental issues have a large impact on children's mathematical achievement. And Fred Smyth, of the University of Virginia, discussed his work showing that unconscious biases can take a toll on the mathematics achievement of stereotyped groups.

The work of both researchers suggests that society can have a big impact on the participation of women in mathematics, and that doing so will have beneficial effects for many groups, not just women.

The discussion of women in the sciences heated up in 2005, when Lawrence Summers, president of Harvard University at the time, hypothesized that the main reason so few mathematicians at top research universities are women is hard-wired differences between the sexes in "intrinsic aptitude" for mathematics, especially at the very high end of the distribution. On many mathematical tests (the SAT, for instance), average scores for boys and girls are similar but the variation in boys' scores is much greater, with more scoring quite poorly and more scoring quite well. Summers guessed this might be indicating that by nature, many more boys than girls are very talented in mathematics.

Mertz and her colleagues have been looking for solid evidence to clear up the debate. To tease out whether the differences Summers observed are driven by environmental or biological influences, Mertz's team examined data across many different countries. If the differences in variability were biologically determined, they should be similar across different cultures. Furthermore, they noted that Summers' argument assumes that scores on the SAT and similar tests are good indicators of the kind of mathematical talent required to succeed at research mathematics, but such tests are not designed to detect extraordinary mathematical gifts. So Mertz's team analyzed data from high-level math competitions, which are indeed designed to detect profound aptitude for mathematics.

The data showed that there are many girls who have great intrinsic ability in mathematics and that cultural and educational factors have an enormous impact on whether this intrinsic ability is identified and nurtured. Furthermore, the team found that the problems that hold girls back from developing their mathematical talents to the fullest affect many American boys as well. Their article, Cross-Cultural Analysis of Students with Exceptional Talent in Mathematical Problem Solving, appeared in the November 2008 Notices of the American Mathematical Society.

Mertz and her colleagues started by analyzing data from the International Mathematical Olympiad (IMO), a very challenging exam taken by middle and high school children around the world each year. The participation rates of girls varied dramatically between countries. The USSR has had 13 different girls on the six-member teams since 1974, whereas the US has had just 3. Furthermore, countries like the USSR and Eastern European countries that have strong math circle programs typically have more girls in the IMO and also tend to do quite well in the competition. Several girls have scored at the very highest levels on the exam. Given that mathematical talent is probably fairly evenly distributed around the world, this data suggests that in the U.S., mathematically talented American girls aren't being efficiently identified and nurtured.

The team found that US participants in the IMO, the USA Mathematical Olympiad and the college-level Putnam Mathematical Competition are often immigrants from countries where mathematical education is considered to be important. Asian-American girls and white girls who emigrated from Eastern Europe are represented in the competitions in proportion to their percentage of the US population. American-born white and minority girls, however, are enormously underrepresented.

A similar pattern was found among mathematics faculty in five top US mathematics departments. Only 20% were born in the US, and of the remaining 80%, many are immigrants from countries in which girls frequently participate in the IMO and in math circles.

American-born white girls aren't the only ones who are suffering. White boys are significantly underrepresented relative to all boys. Mertz points to differences in attitudes toward education and mathematics as key. "Whites tend to view performance as a measure of innate ability, whereas Asians tend to view it as a result of working hard," she say. "Whites view it as OK to be poor at math, whereas Asians view math as a vital skill."

Mertz and her team extended this work in the June 1 issue of the Proceedings of the National Academy of Sciences. The team found women are underrepresented in the Harvard faculty even relative to the predictions of Summers' theory. Furthermore, the ratio between the variability in boys' scores and girls' scores vary widely across countries and between cultures within countries, suggesting the variability is strongly affected by cultural factors rather than being innate.

The impact of implicit attitudes on math performance

Several months after Larry Summers' notorious assertion, he made a far less well-known statement. "Any of us who think that we can for ourselves judge whether we are biased or not are probably making a serious mistake," Summers said at the National Symposium for the Advancement of Women in Science in 2005. "So we all need to think about what we can learn from data about our own unconscious biases and what we can do structurally to overcome these biases."

In the intervening months, it seems that Summers had learned of Fred Smyth's work.

Even people who genuinely believe that sex and race don't affect mathematics ability may nevertheless have unconscious biases, Smyth and his colleagues have shown. Furthermore, these cultural biases can take a toll on the mathematics achievement of girls and other stereotyped groups.

Smyth has demonstrated this in many different ways. In a recent study, he gave teachers a case study of a rising middle school student who was sometimes given a female name and sometimes a male one. The elementary school had recommended that the student skip the 6th grade honors class in math and go into the 7th grade class. The student and parents were enthusiastic, though the parents wondered whether there might be academic or social pitfalls. The teachers in the study were asked to make a recommendation to the parents. Female teachers, he found, were slightly more likely to recommend that girls move ahead than boys. Male teachers, however, recommended that the boys move ahead about twice as often as the girls.

Smyth has also found that when women are reminded of their gender before a test they want to do well on, their concern about fulfilling a negative stereotype can make them do worse. This phenomenon is known as "stereotype threat." Some researchers gave Asian American women a math test, and before it, they reminded the women either of their gender (by asking them about their preferences regarding single-sex or co-ed dorms), or their race (by asking them about their family history), or neither. The women who had been reminded of their gender did worse on the test, and those who had been reminded of their race did better. Similar experiments have shown that when the suggestion was made to women that biological differences caused women to perform less well in math, their performance declined, and when the suggestion was made that environmental effects made the difference, their performance improved. Women's performance is also depressed when they are in the minority in a group of students.

Smyth and his colleagues have devised a clever web-based test called the Implicit Association Test which measures an individual's level of cognitive dissonance between gender and science. The test is available online at http://implicit.harvard.edu. The test starts out straightforwardly: The computer flashes words on the screen, and the person being tested sorts them as quickly as possible into categories, putting, for example, words for men (like "father") on the left and words for women (like "mother") on the right. The person next sorts words for science from words for humanities. Then the test gets a bit harder: The computer flashes up words either for genders or for fields, and the person has to put words that are either for men or for science on the left and words for women or humanities on the right, as fast as possible. The person then repeats the test with a switch, so that men or humanities are on the left and women or science are on the right.

Typically, people are significantly slower when putting science words with female words than when putting humanities words with female words. Remarkably, this effect is significantly stronger in male scientists than in the general population (and in female scientists, it's weaker).

Smyth has data on results of the Implicit Association Test from countries around the world. Countries with higher levels of bias on the test, he has found, tend to have a greater difference between the scores of girls and boys on an international mathematics test.

Bias can be changed through mindfulness. People who are exposed to ideas or images that counter the stereotypes subsequently show less bias on the Implicit Association Test, Smyth has found. Furthermore, female students who themselves show less bias are less susceptible to stereotype threat.

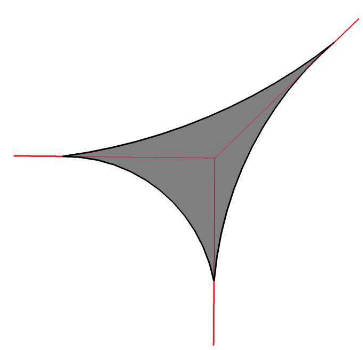
Smyth says that these results suggest that girls' low participation in math and science is strongly related to environmental cues.

Tropical Geometry

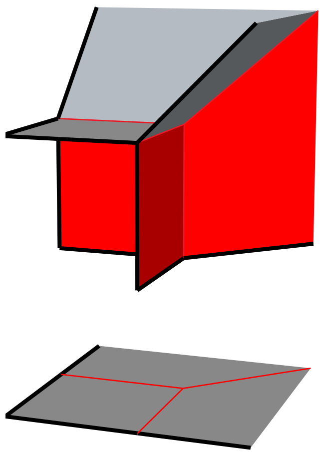
(continued from page 1)

In this framework, the equations that define a complex projective variety "tropicalize" to the corner loci of piecewise affine-linear functions over $\mathbb{R}$, which in turn has become a semifield with respect to addition (in place of multiplication) and pairwise maximum (in place of addition). The approach also anticipates Mikhalkin's geometry of *tropical manifolds*. A key ingredient is the notion of *modification*, which allows one to think of the tropicalizations of (abstractly) isomorphic algebraic varieties as essentially equivalent tropical objects.

An alternative view of tropical geometry, as developed by the Sturmfels school, comes from the theory of Gröbner bases. This works as follows. Let $\mathbb{K}$ be a field equipped with a nonarchimedean valuation $\nu : \mathbb{K} \to \mathbb{R}$. A variety X embedded in $\mathbb{P}^n_{\mathbb{K}}$ is defined by an ideal I in the coordinate variables of $\mathbb{P}^n$. Any choice of weights w on the coordinate variables singles out a specialization of X to a variety defined by the initial ideal $\mathrm{in}_w(I)$ of I with respect to w. The tropicalization, $\mathrm{Trop}(X)$, of X is defined to be the set of weights w for which $\mathrm{in}_w(I)$ contains no monomials. From this definition, the polyhedral structure of $\mathrm{Trop}(X)$ follows easily. On the other hand, a fundamental result of Speyer and Sturmfels establishes that $\mathrm{Trop}(X)$, which is a subset of $\mathbb{R}^n$, for some $n > 0$,



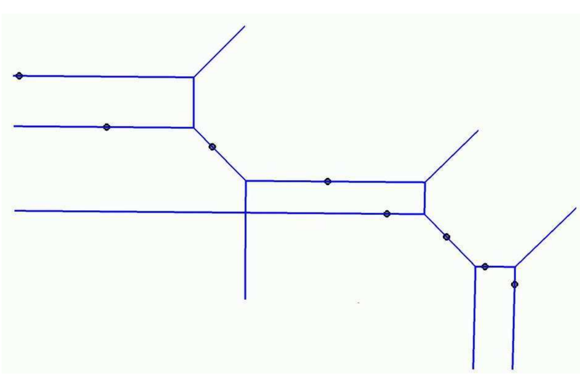
Logarithmic degeneration to a line in $\mathbb{TP}^2$



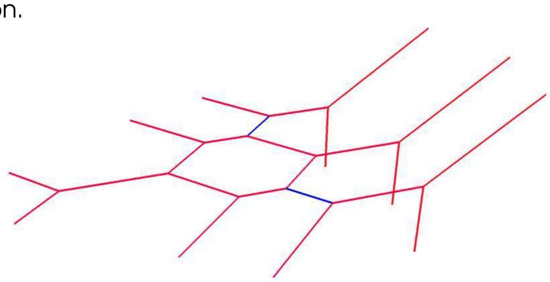
Modifying $\mathbb{TP}^2$ (below) along a line to a plane in $\mathbb{TP}^3$.

is the set of valuative n-tuples $\nu(X)$. According to this point of view, $\mathrm{Trop}(X)$ is a polyhedral shadow of X. The work of Speyer, Sturmfels, and Williams on tropical Grassmannians exemplifies the Gröbner-based approach.

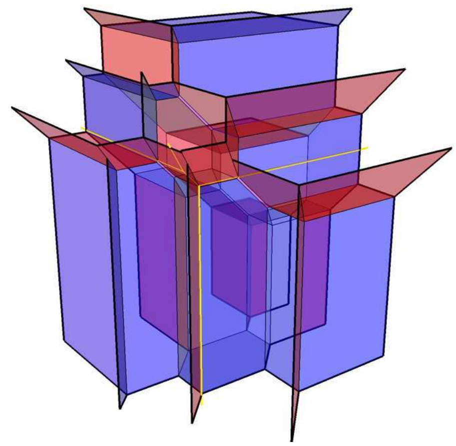
The miraculous property of $\mathrm{Trop}(X)$, however it is constructed, is that often it obeys the same basic theorems as X, once these have been appropriately parsed. The prototypical result along these lines is Mikhalkin's correspondence theorem relating plane curves (of classical and tropical types, respectively) of degree d and genus g that pass through $3d + g - 1$ points in general position. Soon after, Mikhalkin and Speyer proved independently (and via different methods) that every zero-tension tree in $\mathbb{R}^n$ is the tropicalization of a rational curve defined over (complex) Puiseux series, provided it has the appropriate number of leaves. The situation in higher genus, however, is significantly more complicated. Similarly, if one replaces the ambient projective space by an arbitrary projective variety, there are subtle obstructions to realizability, even in the most seemingly innocent cases. For example, in contrast with the situation over the complex numbers, Vigeland exhibited smooth cubic surfaces in tropical three-dimensional projective space with infinitely many lines!



Rational cubic plane curve through eight points in general position.



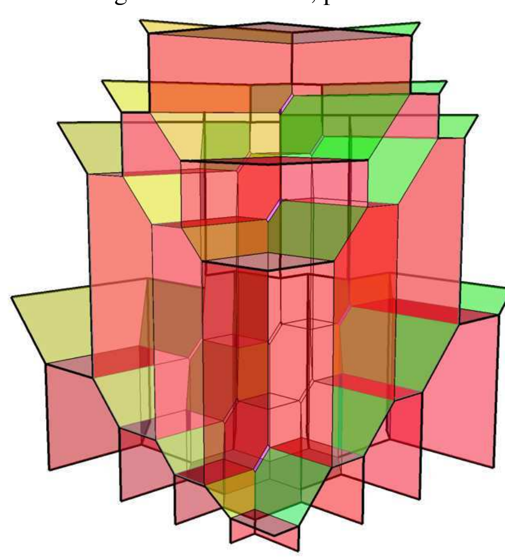
Cubic space curve. The curve is well-spaced because the two blue segments are of the same length.



A line on a cubic surface.

Important recent progress on realizability was the focus of ongoing discussions over the course of the tropical semester. Katz and Payne proved that associated to any effective simplicial fan, there is an algebraic realization space whose points correspond to irreducible reduced schemes that tropicalize to that fan. Brugallé and Mikhalkin proved a generalization for curves of arbitrary genus of a result of Speyer's "well-spacedness" result for elliptic curves, which gives a necessary criterion for realizability. And during the semester, Katz discovered a geometric explanation for the non-realizabilty of Vigeland's moving lines on cubic surfaces.

Realizability is a crucial problem for tropical geometry, since it is the thread that allows inferences about algebraic varieties to be made on the basis of a description of their tropical counterparts. For example, Gibney and Maclagan show how to reduce Fulton's conjectural description of the effective cone of $\overline{\mathcal{M}}_{0,n}$, the moduli space of n-pointed rational stable curves, to a combinatorial problem with an algorithmic solution, provided one can prove that



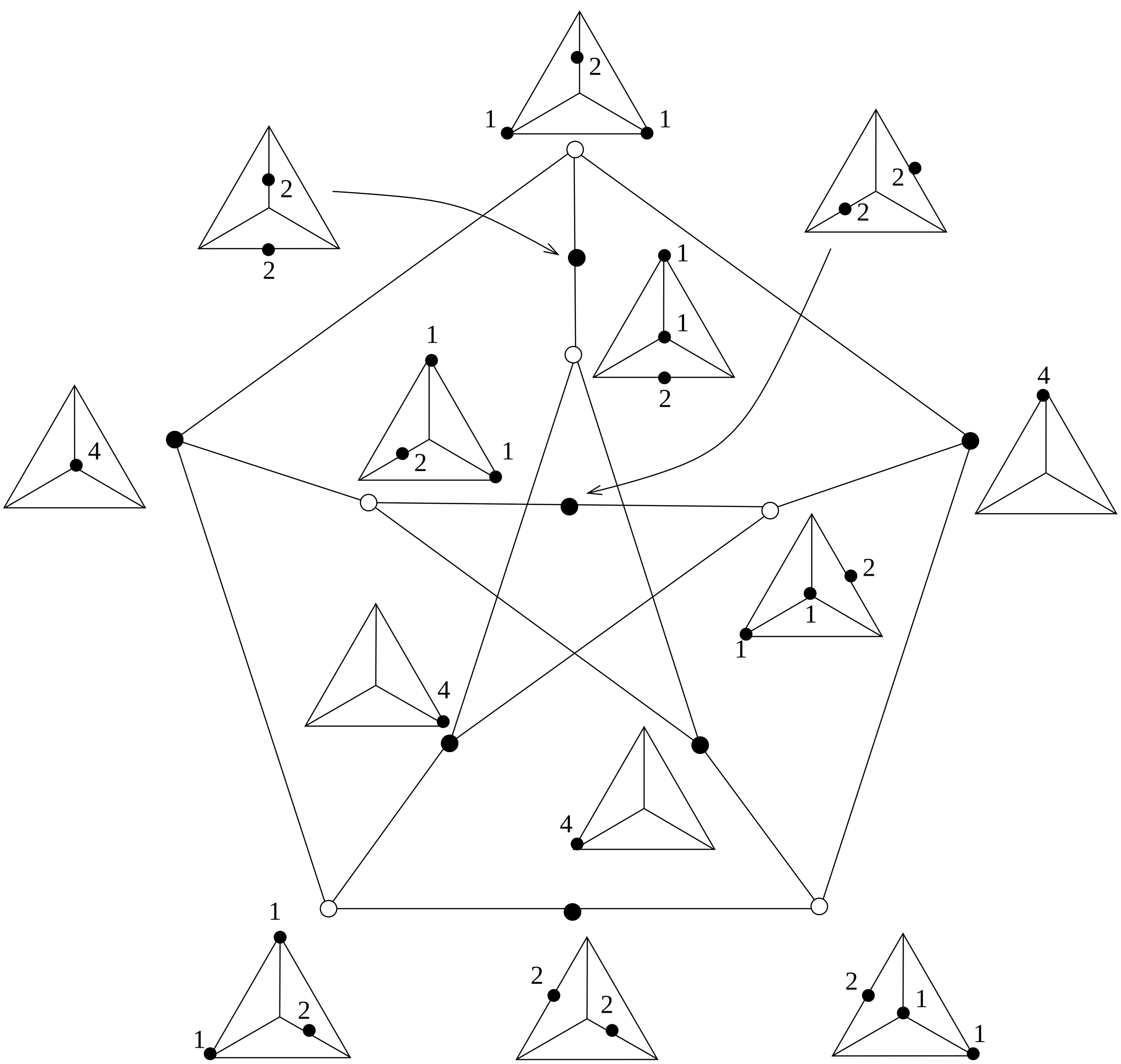
A quartic surface decomposed as a cubic glued to a modified $\mathbb{TP}^2$.

generators of the tropical effective cone lift to generators of the cone over $\mathbb{C}$.

Realizability also intervenes in an important way in the tropical analogue of Clemens' celebrated conjecture that all rational curves on a general quintic threefold are rigid. Namely: is there *some* tropical quintic threefold all of whose zero-tension subtrees are tropically rigid? If so, does this imply rigidity for rational curves on the corresponding threefold over Puiseux series? (One suspects the answer is yes.) And if not, can one show that nonrigid subtrees do not arise as the tropicalizations of rational curves over Puiseux series? A confirmation of Clemens' conjecture would give an enumerative justification for the predictions of mirror symmetry; groundbreaking recent work of Gross and Siebert gives a tropical construction of the latter phenomenon. Most notably, Gross and Siebert show how to reconstruct families of complex curves given seed data consisting of a tropical curve equipped with a log structure.

A related line of inquiry is the study of linear series on tropical varieties. The most basic case is that of curves, where it is natural to ask for a tropical analogue of the classical Brill–Nöther theory of linear series on algebraic curves. To date, the main progress on these questions has been obtained by Baker–Norine, Gathmann–Kerber, and Mikhalkin–Zharkov, who proved theorems of Riemann–Roch type. Baker and Norine showed, notably, how to interpret tropical linear equivalence vis-à-vis chip-firing moves on metric graphs. In recent work, Haase, Musiker, and Yu coordinatize the Baker–Norine construction and obtain explicit models of complete linear series as tropically convex subspaces of (tropical) symmetric products. Their work may be viewed as a first step towards a geometric theory of tropical linear series, based upon the study of canonically embedded tropical curves.

The topics discussed above represent, of course, only a fraction of the recent progress in tropical geometry. Notably absent is any discussion of matroids, which not only play a crucial rôle in understanding the geometry of the tropical Grassmannian, but are also of interest as combinatorial objects in their own right. This reflects the geometric bias of the author. Hopefully, the reader will retain a sense for the broad relevance, and vitality, of tropical geometry, which continues to surprise us with connections to more traditional areas of mathematics.



The canonical complex of the complete metric graph on 4 vertices, with all edges of equal length.

Puzzles Column

Elwyn Berlekamp and Joe P. Buhler

1. You overhear a conversation in which it becomes known that one person has two children, one of them a boy who was born on a Tuesday. What is the chance that the other child is also a boy? (The answer depends on how one interprets the relevance — or lack thereof — of the reference to Tuesday. So an interesting meta-question is: how many plausible contextual backgrounds to the conversation can you specify that lead to different answers?)

Comment: This is a riff on a favorite Martin Gardner problem; see for example Chapters 14 and 19 of his *Second Book on Mathematical Puzzles and Diversions* (1961). (His more recent *Colossal Book of Mathematics* presents other probability puzzles in which several natural interpretations are possible.) The particular version given here is attributed to Garry Fosbee on Ed Pegg's charming Math Puzzle website.

2. Consider the one-person backgammon position in which all 15 checkers are at the one-point. Each roll of the dice either bears off 4 checkers (if the roll is a double) or 2 (otherwise). What is the probability that the final roll will be a double? What is the limiting probability as the number of checkers goes to infinity?

For the backgammon-challenged: A roll of the two dice is called a double roll when the two faces are the same. When the checkers are at the one-point their next move is to go off the board, so the rules are simple and as stated. If there is only one checker left, two dice are still rolled, so even in that position a game might end with a double roll. (In a general backgammon position, a non-double roll allows the player to move checkers of her choosing by the face values, each die counting twice in the case of a double roll.)

Comment: A more elaborate version of this question, aimed at a lower bound on the probability that a game of backgammon ends with a double roll, appeared on the IBM puzzle web page "Ponder This" in July 2009; see domino.watson.ibm.com/Comm/wwwr_ponder.nsf/challenges/July2009.html

3. Let $\mathbb{Z}^3$ denote the set of points in 3-space whose coordinates are integers (forming a cubical lattice of points). Show that it is not possible to embed a regular icosahedron in that set, i.e., find 12 elements of $\mathbb{Z}^3$ that are the vertices of a regular icosahedron.

Now let L be an arbitrary lattice in 3-space, i.e., the set of integral linear combinations of three linearly independent vectors. Is it possible to embed a regular icosahedron in L?

Comment: Inspired by a preprint on the arXiv by Ionascu and Markov, arXiv:0910.1722.

4. Let n be a positive integer of the form $n = 6k+1$.

(a) Show that $3((n+1)^{n+1} - n^n)$ is divisible by (n^2+n+1).

(b) Show that $3((n+1)^{n+1} - n^n)$ is divisible by $(n^2+n+1)^2$.

Comment: We owe this to Rich Schroeppel, who found it empirically, along with several similar results.

5. Nathan and Peter are playing a game. Nathan always goes first. The players take turns changing a positive integer to a smaller one and then passing the smaller number back to their opponent. On each move, a player may either subtract one from the integer or halve it, rounding down if necessary. Thus, from 28 the legal moves are to 27 or to 14; from 27, the legal moves are to 26 or to 13. The game ends when the integer reaches 0. The player who makes the last move wins. For example, if the starting integer is 15, a legal sequence of moves might be to 7, then 6, then 3, then 2, then 1, and then to 0. (In this sample game one of the players could have played better!)

(a) Assuming both Nathan and Peter play according to the best possible strategy, who will win if the starting integer is 1000? 2000?

(b) As you might expect, for some starting integers Peter can win and for others Nathan can win. If we draw the starting integer at random from all the integers from 1 to n inclusive, we can consider the probability of drawing a position from which Nathan can win. This probability will fluctuate as n increases, but what is its limit as n tends to infinity?

Comment: Due to Mark Krusemeyer, this appeared on the 2009 MathCamp qualifying quiz.

The CME Group – MSRI Prize in Innovative Quantitative Applications

The fourth annual CME Group-MSRI prize was awarded on September 17, during a celebration held at the CME Group headquarters in Chicago.

This award was established in 2006 by the CME Group and MSRI to recognize an individual or a group for originality and innovation in the use of mathematical, statistical or computational methods for the study of the behavior of markets, and more broadly of economics. The recipient receives a commemorative bronze medallion and a cash award of $25,000.

Recipients of the award so far:

2006 **Steven A. Ross** 2007 **David Kreps** 2008 **Lars Peter Hansen** 2009 **Sanford Grossman**

Great Circles 2009

Conference gathers veterans of math circles, where kids learn to love math

Julie Rehmeyer

Math is freedom. Math is joy, math is creativity, math is play.

Mathematicians know that as simple fact, but most people would greet that statement with a blank stare. So how can mathematicians get the word out and show the world what mathematics really is?

Math circles are one way. Across the country, people who have come to know the beauty of mathematics are meeting with kids in living rooms or borrowed classrooms. In these math circles, children grapple with tantalizing mathematical questions, throw out ideas with their friends, and explore a universe of pure thought.

"A math circle is a place where people motivate each other to learn mathematics by sharing their love and enjoyment of the subject," says Mark Saul, who taught math at the Bronx High School of Science and also led extracurricular math circles for many years.

Saul's definition is deliberately flexible, because the spontaneous parallel emergence of math circles has led to tremendous variation. In some math circles research mathematicians lecture, while in others there is no explicit instruction at all. Some explicitly prepare kids for high-level math competitions, while others discourage competition in order to cultivate a collaborative atmosphere. Some borrow heavily from the Eastern European tradition of math circles and some are home-grown inventions. Some are led by mathematicians, some by grade school math teachers, and some by parents with little formal mathematical training. Some are focused on exceptionally mathematically talented kids, some make special efforts to make it fun for kids of all abilities, and others deny the notion of mathematical talent entirely.

What they have in common, though, is that kids play with mathematics, learn to love it, and develop their ability to solve problems and create mathematics for themselves. "Math circles are like a chorus," Saul says. "If you go to a music school, the kids are singing in the hallways. They're irrepressible. They love what they're doing. This is what you can create with a math circle."

There are now over fifty active math circles in the U.S., but that is far from covering the entire country. In April MSRI held a conference, made possible by a grant from the S. D. Bechtel, Jr. Foundation, to bring those involved in math circles together to swap ideas, cheer one another on, introduce newcomers to the concept, and strategize about how to make math circles far more prevalent than they are today. Here are the stories of a few of these math circle veterans.

The Math Circle in Boston

Bob and **Ellen Kaplan** have run a math circle in Boston since 1994, catering to any student who thinks it might be fun. Everyone, they say, has an innate capacity and talent for mathematics, and they aim to create an environment that will bring that out in any interested student.

The Kaplans' method is to pose a mathematically rich, accessible and tantalizing question and then let the conversation unfold. They describe themselves as their students' sherpas, more experienced guides who can carry gear and suggest routes. The climbing, however, is done by the students themselves. The students will refine the question, decide for themselves what's interesting and what's not, toss around ideas and discard them. The key ideas always come from the students themselves. To create a collaborative atmosphere, the Kaplans discourage competition in their classes, though some of their students do participate in math competitions.

Bob Kaplan demonstrated the method on a group of nine four- and five-years olds (though with such young kids, he recommends limiting it to six). Once the kids had filed into their chairs, Kaplan said, "Hi, my name is Bob. I was wondering, how big is big?"

"Very, very big!," the children cried out.

"And how big is that?" "Very very very big!," a little girl called out. "A truck!," a boy hollered. Another boy jumped out of his chair and held his hand over his head: "This big!"

"How big is that?," Kaplan asked. "Five feet tall?" "No, two." The boy plopped down in his chair with satisfaction.

Kaplan smiled merrily. "Two is pretty big. Anyone know anything bigger than two?"

"Ten thousand!"

"That's a very big number," Kaplan said admiringly. "Is it bigger than two? How do you know?"

"Because it's way bigger!"

And again: "How do you know?" Shrug. So Kaplan tried a new tack: "I think 10,000 is the biggest number there is."

"No. There's a million." Another girl called out, "A trillion!"

"A trillion! Maybe that's the biggest number, then," Kaplan said.

"No, the numbers just keep going on and on," said one child.

"No, the numbers stop," said another.

"Hmm, which one is it?" Kaplan asked. "Do the numbers go on and on, or do they stop? How could we know for sure?"

Now the children had gotten their teeth into an important mathematical question, and for the next 30 minutes, they turned it one way and then another. When their four-year-old attention wavered, Kaplan guided them back, or moved the conversation in a new direction, or interrupted with a brief mathematical game. They wrestled with ancillary mathematical questions along the way: Are there numbers between 0 and 1? How might one split a pizza into nine equal parts? Is it reasonable to give a name to a number? Is infinity a number?

A few minutes before the end of the session, Kaplan wondered again about whether the numbers stop or not. One of the children came up with a key insight: Whatever number anyone named, he could add one and get a bigger number. Therefore, they could be sure the numbers never stopped.

On that triumphant note, the session ended.

The Albany Area Math Circle

Mary O'Keeffe had read about math circles long before she ever considered one of her own. "I thought, 'I could never do that,'" she said. "I somehow thought you had to be divinely anointed."

But when her daughter Alison was bored in math in third grade, the school gave Alison a computer with a drill program and sent her off to work on her own — in a supply closet. O'Keeffe was outraged. She figured, "I can do better than a supply closet!"

She recognized she'd have to create opportunities on her own for Alison to do math in a social setting. She started home-schooling Alison, and the two of them went to local elementary schools and volunteered to lead math groups for bored troublemakers.

One day, she brought in a pan of brownies and asked the kids, "How can we keep eating these brownies but make them last forever?" Eventually, the kids figured out themselves that they could eat half the brownies the first day, half the remainder the second day, and so on, effectively creating a series adding up to 1. Then they ate the brownies — all of them at once.

Before long, schools were clamoring for O'Keeffe and her daughter. When Alison reached middle school, the two of them started coaching teams for the Math Counts competition. O'Keeffe then turned to books that supplied challenging problems for Math Counts, but she hit a glitch: Despite her undergraduate degree in math, she couldn't solve many of the problems. Alison could solve more than she could, but even so, O'Keeffe found herself cutting up lists of problems, picking out the ones they could solve and then copying the reassembled versions.

Eventually she realized that was silly. She began instead just being honest with the kids and letting them know when she didn't know the answer. She found that the honesty was actually valuable in creating a positive culture among the kids, one in which they weren't afraid to not know or be wrong.

When Alison reached high school, they couldn't find enough kids at any one school to field a team for a competition. But they found a collaborator: **Mukkai Krishnamoorthy**, a computer science professor at the Rensselaer Polytechnic Institute, whose son Raju was a year younger than Alison and loved math. In 2001, the partners found a room to use at the Rensselaer Polytechnic Institute and had the first meeting of the Albany Area Math Circle.

The group meets once a week, and the students spend the first 75 minutes working individually on a list of very challenging problems, one far too long and difficult for any one person to successfully complete them all. The students then take a break for pizza, and then they assemble into groups to work further on the problems. To make this practical, the math circle meets in a very large room with tables and chairs that are easy to rearrange.

The Albany (NY) high-school math circle has two annual outdoor meetings. This one happened in 2006, at River Road Park in Niskayuna, NY. O'Keefe is second from the left, and her two daughters, Catherine and Alison Miller, are sitting near the top of the geodesic dome in purple and light blue shirts. Krishnamoorthy is second from the right. The photo includes group "alumni" who are now in college but were home on vacation, as well as some younger students from circles for middle schoolers.

O'Keeffe finds that her main role is to nurture the sense of community among the students. She finds students who seem disconnected and introduces them to a new group. She'll match a student who feels discouraged with a new member of the math circle, thereby encouraging both members of the pair. She also emphasizes the value of making mistakes. On their website, she has a Hall of Fame where she lists the students who do the most to inspire and help others before she lists the competition winners.

O'Keeffe feels somewhat ambivalent about competitions, but she finds that they are motivating and fun for the students. And indeed, the Albany Area Math Circle has produced 10 USA Math Olympiad qualifiers (only 500 students across the country qualify each year). Five of their students have gotten a perfect score on the American Mathematics Competition test.

O'Keeffe's daughter Alison has now finished college and will soon begin a graduate program in math, but O'Keeffe continues to run the math circle.

"Now everyone thinks I was anointed from above," O'Keeffe says. "But really, a math circle is like a sewing circle. Bring your sewing stuff to my house and let's sew together."

The Berkeley Math Circle

Growing up in Bulgaria, **Zvezda Stankova** always knew that being in a math circle was cool. Just like the athletic kids joined a sports team, the smart kids joined a math circle or a physics circle or a poetry circle. It was fun, plus it was the first step to a life in math or science, since most of the great mathematicians and scientists in Eastern Europe had grown up with math circles.

Even cooler, the kids who were best at it got to travel to competitions around the world. In 1987 and 1988, Stankova herself went to the International Mathematical Olympiad, where six top students from each country around the world have a few hours to tackle a small number of very hard problems.

At the weekly meetings of the Berkeley math circle, different teachers give talks on a wide range of subjects, covering geometry, number theory, topology, probability, game theory, and more. "We talk about the beauty of math in topics that are not usually covered at school," says Stankova, who is now a math professor at Mills College. "This program is really for talented, bright kids who want to be challenged and learn the depths of mathematics."

Many kids from the Berkeley Math Circle have gone on to win prizes in various math contests. Several have made it to the International Math Olympiad, for which only six U.S. students are chosen each year from about half a million contestants. Berkeley Math Circle participants have won nine medals in the international competition.

Stankova dreams of a math circle at every college and university in the U.S. She imagines a professor receiving time released from other responsibilities to organize the circle. Undergraduates could participate in the circles for course credit alongside the grade-school students. Students who aren't members of the university would pay a modest fee, and the department would provide secretarial and computing support as part of its outreach efforts. This would provide a sustainable infrastructure that would benefit the math community as a whole.

A Letter from the Director

It is a great pleasure for me to take this opportunity to thank the many individuals, companies, foundations, and others listed on the Donor Roster that have contributed to MSRI over the past fiscal year (August 1, 2008 through July 31, 2009).

In the course of leading an organization with a multi-million dollar annual budget, I've faced many challenges during my Directorship so far. Contributions from the supporters listed on the Donor Roster have helped meet these challenges. Whether it was the need to bring in a top researcher to anchor a program, to support a young researcher for whom participation in the program would be a career-changing event, or to fund other critical aspects of MSRI's mission, the donors whose names appear on the next two pages have helped me to provide for those critical opportunities, even when we could not cover it from our core funding.

Other challenges are not as proximate but call out loudly nonetheless. Needs of the general public and our need to share with them the beauty and majesty of mathematics are addressed by our amazing outreach programs. The elementary, middle, and high school students who hunger for a deeper experience of mathematics than their teachers alone can provide is met by Math Circles, Olympiads, and Festivals, all made possible by those on the Donor Roster.

Gifts of endowment and planned gifts through our Gauss Society have helped us make progress in assuring that MSRI's future is bright and certain.

Other gifts, such as those to our mug fund (which allows us to provide a hand-thrown personalized coffee mug to each member), help facilitate social interactions among our members, making research at MSRI even more rewarding and stimulating.

For these acts of generosity to the mathematics community at MSRI, I am deeply grateful. I am sure to face many more challenges in the year ahead and, with your support, hope to find creative solutions that broaden and deepen the mathematical experience of those touched by MSRI.

Sincerely,

Robert L. Bryant, Director

P.S. We have attempted to provide an accurate listing of our donors in the Donor Roster. If your name is missing or listed incorrectly, please call our development office at 510-643-6056, inform us of the mistake, and accept our sincere apology.

2008-2009 MSRI Donor Roster

Recognizing MSRI's donors making gifts from August 1, 2008 through July 31, 2009

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Fibonacci Donors $34 to $89

Both the upper and lower end of this range are Fibonacci Numbers. 34 is also the magic constant of a 4 by 4 magic square.

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257 is a Fermat number, and is equal to 2 to the 2 to the 3 plus 1.

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729 is a number important to Plato and a cube that is the sum of three cubes.

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Ramanujan Donors $730 to $1,729

1,729 is the number of Hardy's taxicab, which, Ramanujan reflected from his sick bed, is the smallest number expressible as the sum of two cubes in two different ways.

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Noether Donors $1,730 - $1,921

1921 is the year of publication of Noether's "Idealtheorie in Ringbereichen" – a landmark paper ushering in the beginning of the field of Abstract Algebra, which became a dominant theme of 20th Century mathematics and flourishes into the 21st century.

Plato Donors $1,922 to $5,040

Plato, in The Laws, suggested that a suitable number of citizens for the ideal city would be that number which contained the most numerous and most consecutive subdivisions. He decided on 5,040, a number with 59 divisors (apart from itself). For purposes of war and peace 5,040 citizens can be divided by any number from 1 to 10.

Jennifer T. Chayes & Christian H. Borgs
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Archimedes Society, continued

Museion Donors $5,041 and above. Named for the recognition event to which its members are invited, Museion, "the Hall of the Muses", was Ptolemy I's institute at ancient Alexandria. Scholars came to study and advance science, and the adjacent library was said to be the greatest in the world at the time.

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The Gauss Society recognizes individuals who are making a planned gift to MSRI through mention in their 403(b) retirement plan, will, or estate plan. Members meet annually in January for the Gauss Society Dinner and Lecture.

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New Postdoctoral Fellowships

The impact of the economic downturn has hit academia hard, causing hiring freezes and cancelled job searches. For mathematics this represented a loss of some 400 positions for recent PhDs. The National Science Foundation (NSF), through its seven mathematics institutes (including MSRI), responded by creating new postdoctoral fellowships. In a new initiative, this partnership of math institutes created 45 postdoctoral positions for young, highly-trained mathematical scientists across the country, known as NSF Mathematical Sciences Institutes Postdoctoral Fellowships. Ten of these were awarded by MSRI.

"We knew that the job market for young PhDs in mathematics was extremely tight this year, but we were astonished by the number and quality of the applicants for these new positions," said MSRI director Robert Bryant. Over 750 applications were submitted for the 45 available slots. "Being able to offer these positions allows us to keep these highly trained people in the workforce," continued Bryant. The program will support postdocs working in a dozen states, in all areas of the mathematical sciences.

Four of the ten exceptional mathematicians awarded the new NSF Postdoctoral Fellowship by MSRI are participating in MSRI programs right now, and got their fellowship for 2009/10 (see below). The other six received one- and two-year fellowships allowing them to pursue their work at several institutions: **Vigleik Angeltveit** will continue his research with Peter May at the University of Chicago; **Scott Crofts** is at UC Santa Cruz for two years to work with Martin Weissman; **Anton Dochtermann** will continue his work with Gunnar Carlsson at Stanford University; **Karl Mahlburg** works at Princeton University with Manjul Bhargava and Peter Sarnak; **Abraham Smith** is at McGill University working with Niky Karman; and **Jared Speck** also works at Princeton University, with Sergiu Klainerman. See details at http://www.msri.org/specials/nsfpostdocs.

Tristram Bogart is one of the four Fellows now at MSRI. He did his PhD at the University of Washington with Rekha Thomas. He studies combinatorial algebra and algebraic geometry. His fellowship will enable him to be at San Francisco State University next academic year, collaborating with Federico Ardila. "I'm excited to spend next year in the Bay Area following up on some of what I'm learning at the MSRI Tropical Geometry program," he said.

Sikimeti Ma'u, originally from Tonga, is a permanent US resident. She is part of the year-long Symplectic and Contact Geometry and Topology Program at MSRI, after which her NSF fellowship will take her to Barnard, to work with the distinguished topologist Dusa McDuff. "It's a really exciting opportunity," said Sikimeti, "to be at the MSRI while so many leading mathematicians in the field will be there and to be mentored by one of them."

Christopher Hillar will work with Fritz Sommer at UC Berkeley's Redwood Center for Theoretical Neuroscience, an interdisciplinary group of researchers working to develop mathematical and computational models for the neurobiological mechanisms in the brain. Hillar expects that his NSF Fellowship will provide him a rich cross-disciplinary research interaction. "I am honored to receive an MSRI NSF Postdoctoral Fellowship," said Hillar, an algebraic geometer. "This award will allow me to explore foundational problems in mathematical neuroscience, and I hope to use this opportunity to engage other mathematicians in this pursuit."

Eric Katz did his PhD at Stanford with Yakov Eliashberg; his interests include tropical and algebraic geometry, enumerative geometry, toric varieties, and relative Gromov–Witten theory. Currently at MSRI, he will return to the University of Texas, Austin to work with Sean Keel. "The Institutes' Postdoc is a great opportunity," said Eric. "Being at MSRI during the Tropical program has given me exposure to a lot of current work in the area and introduced me to a lot of other researchers."

Besides MSRI, six other NSF-funded institutes led this initiative to create and employ postdoctoral positions: American Institute of Mathematics, Institute for Advanced Study, Institute for Mathematics and its Applications, Institute for Pure and Applied Mathematics, Mathematical Biosciences Institute, and Statistical and Applied Mathematical Sciences Institute.

David Eisenbud

Left to right:
Director Robert Bryant
Postdoctoral Fellows:
Sikimeti Ma'u, Christopher Hillar, Eric Katz, Tristram Bogart
Deputy Director Hélène Barcelo

Summer Graduate Workshop: Toric Varieties

The MSRI summer graduate workshop on toric varieties, organized by David Cox of Amherst College and Hal Schenck of the University of Illinois at Urbana-Campaign, took place June 15–26, and brought together a diverse group of 45 participants, ranging from first- through fifth-year graduate students, with backgrounds in combinatorics, algebraic and symplectic geometry, and commutative algebra. Toric varieties are a class of algebraic varieties (roughly speaking, objects which look locally like the zeroes of a system of polynomial equations) which lie at the interface of geometry, combinatorics and algebra. The class of toric varieties is both large enough to include a wide range of phenomena and concrete enough to provide an excellent computational environment. This atypical combination leads to applications in many other fields including string theory, coding theory, approximation theory and statistics. Toric varieties also provide a wonderful vehicle for teaching algebraic geometry.

Geometrically, a toric variety is an irreducible algebraic set in which an algebraic torus forms a dense open subset, such that the action of the torus on itself extends to an action on the entire set. Combinatorially, a normal toric variety is determined by a fan; the cones in the fan yield affine varieties and the intersection of cones provides gluing data needed to assemble these affine pieces together. Algebraically, an embedded toric variety corresponds to a prime binomial ideal in a polynomial ring. More generally, a toric variety can be described by a multigraded ring together with an irrelevant ideal. The importance of toric varieties comes from this dictionary between algebraic spaces, discrete geometric objects such as cones and polytopes, and multigraded commutative algebra.

Because of the wide range of backgrounds, the workshop had a very intense schedule. In the evenings, there were background lectures on basic material in algebraic geometry (ranging, for example, from valuation rings to vector bundles to sheaf cohomology). Each morning, there were two one-hour lectures on interpreting algebro-geometric concepts in the toric setting. After lunch, participants were presented with several different sets of problems, ranging from very computational (compute the Picard group of a Hirzebruch surface) to more theoretical (prove a lemma stated during the morning lecture). Participants broke up into small groups of six or seven people, helped when needed by the organizers and two very able TAs (Dustin Cartwright and Daniel Erman) from Berkeley. At the end of the afternoon, the groups presented their results to the whole workshop.

During the latter part of the second week, three guest speakers spoke on topics related to toric geometry: David Eisenbud on the cone of betti tables; Matthias Beck on normality and semigroups; and Sam Payne on toric vector bundles. Participants really enjoyed seeing research talks on topics they had just studied. Here are some of their comments:

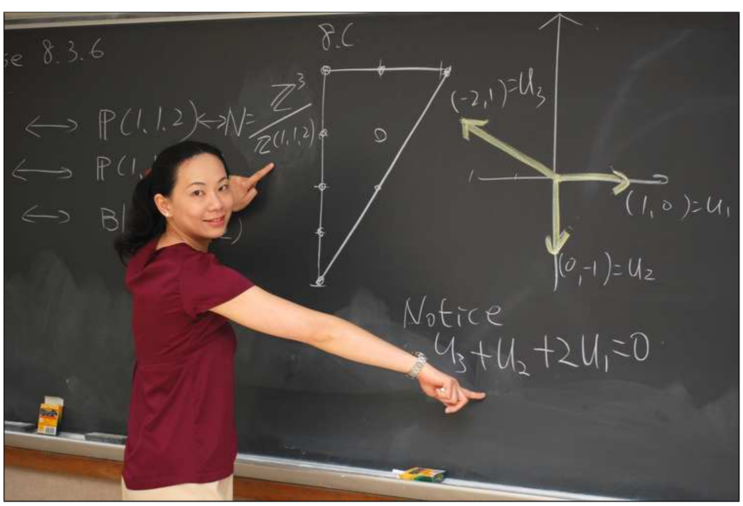
“The workshop was a truly amazing experience. The only way to improve it would be to make it longer!”

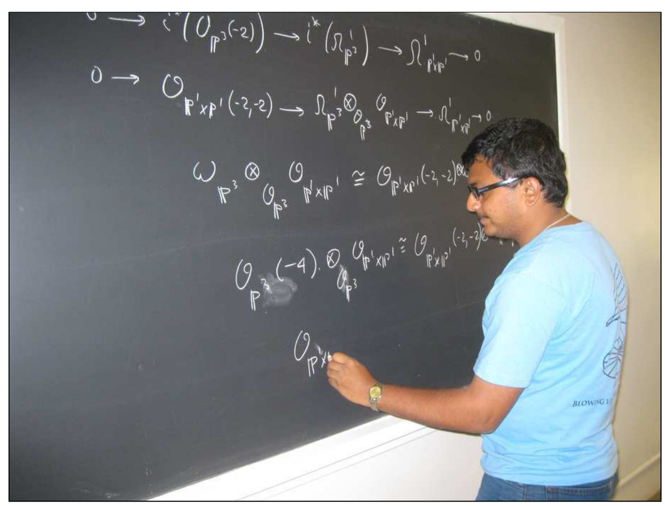
“I can’t believe how much I learned in these short two weeks.”

“Excellent workshop. The problem sessions and presentation setup were very conducive to working together and understanding the material. Intensive but also fun.”

“The format of the workshop, although incredibly intensive, was very effective. Although there was no way for me to have digested everything, I learned a lot. Also, I really enjoyed the problem sessions because it encouraged us to meet each other and socialize.”

“The morning lectures gave us the big picture. The afternoon problem sessions filled in the details of the picture; I particularly enjoyed the group work. The evening lectures helped prepare us for the next day’s topics. This was an awesome experience.”





Kuei-Nuan Lin (Purdue) and Swarnava Mukhopadhyay (UNC) at the summer workshop. Workshop lectures are available on streaming video at http://www.msri.org/calendar/sgw/WorkshopInfo/455/show_sgw. A draft of the forthcoming AMS book *Toric Varieties* by Cox, Little and Schenck is available at http://www.cs.amherst.edu/ dac/toric.html.

Symplectic and Contact Geometry and Topology

(continued from page 1)

Fundamental problems were solved, and many more exciting questions and connections with other areas of mathematics emerged.

A year-long (Fall 2009 and Spring 2010) program is underway at MSRI. Organized by Yakov Eliashberg (Stanford University), John Etnyre (Georgia Institute of Technology), Eleny Ionel (Stanford University), Dusa McDuff (Barnard Colege, Columbia University) and Paul Seidel (MIT), it will further stimulate the now mature and flourishing field of symplectic and contact geometry and topology.

The beginnings

Symplectic and contact geometry are old subjects, which originated as a geometric language for classical mechanics and geometric optics. At the beginning of the twentieth century Henri Poincaré showed that the three-body problem is non-integrable. He also realized that in order to answer qualitative questions about mechanical systems, such as long-term behavior, stability, and the existence and number of periodic orbits, one needed to develop new tools.

For example, he showed that a particular geometric problem about the number of fixed points of an area-preserving transformation of an annulus has consequences about the existence of periodic motions in the so-called restricted three-body problem in mechanics. The geometric problem was solved by G. Birkhoff, a few years after Poincaré's death. However, not until the mid-sixties, when V. I. Arnold formulated his famous conjectures generalizing the Poincaré–Birkhoff theorem, did these ideas receive serious further development.

The field underwent a real renaissance in the mid-1980s, from which symplectic and contact topology were born, and the key technique of holomorphic and pseudoholomorphic curves was introduced to symplectic geometry by M. Gromov.

Holomorphic curves were originally used to address the Arnold conjectures concerning classical questions on the existence of fixed points of symplectomorphisms and double points of Lagrangian submanifolds. They quickly became key tools throughout symplectic and contact geometry, and then took on a life of their own with the advent of quantum cohomology and Gromov–Witten invariants, which, in turn, led to surprising connections with enumerative algebraic geometry and string theory. Holomorphic curves have been particularly effective in low dimensions, where many subtle topological problems have been illuminated by symplectic and contact topology.

Different Floer homology theories for 3-manifolds (notably Heegaard homology) are essentially symplectic geometric creatures. Taubes' work related Seiberg–Witten invariants of symplectic four-manifolds with appropriety redefined Gromov–Witten invariants. Symplectic topology found many interesting applications in dynamics, in the theory of periodic orbits, and via a bi-invariant metric on the group of symplectomorphisms discovered by Hofer. The geometry of this metric and related geometric objects, now called Hofer geometry, became an important tool in dynamics.

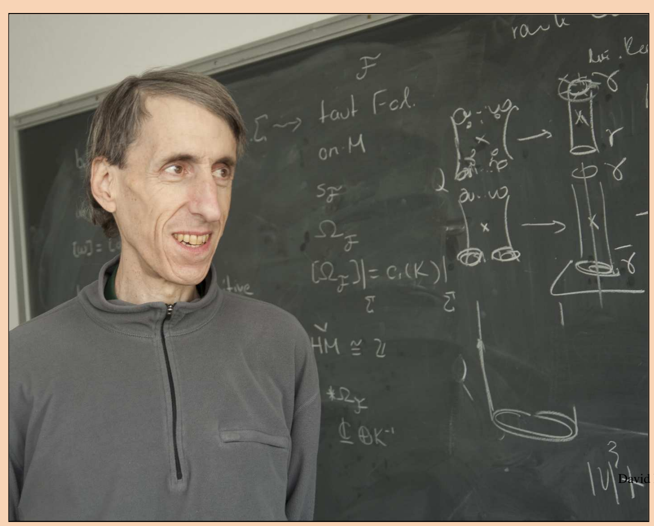
The 2009/10 program at MSRI

The current program began in August 2009 with a Summer Graduate Workshop followed by the Connections for Women workshop. In fact the two workshops had a joint morning session consisting of two survey lectures, intended both to sum up the work of the previous two weeks for the graduate students and provide an interesting survey of the area for the newcomers. There followed an Introductory Workshop on Symplectic and Contact Geometry and Topology, which consisted of a coherent lecture series on Floer and symplectic homology, symplectic field theory, the theory of Lefschetz fibrations for symplectic and open book decompositions for contact manifolds, and applications of symplectic topology to dynamics. There was also active discussion of open problems and main goals for the special year. All three workshops were designed to introduce participants to current developments in the area and to set the main goals for the year-long program.

Cliff Taubes

Clifford Henry Taubes of Harvard University is visiting MSRI for this entire academic year to participate in the program on Symplectic and Contact Geometry and Topology. He works in nonlinear geometric analysis, and his research has produced striking advances in our understanding of how tools such as the Seiberg–Witten equations and pseudoholomorphic curves can be used to answer fundamental questions in low-dimensional topology. His celebrated work relating the Seiberg–Witten invariants to Gromov–Witten invariants has given us deep insight into the topology of four-manifolds. By developing techniques to cover singular symplectic structures, he has shown how his work can be applied even to non-symplectic manifolds. His recent work proving the Weinstein conjecture for contact vector fields in dimension three has generated great excitement.

For his enormously influential work, Taubes received the National Academy of Sciences Mathematics Award in 2008 and shared the 2009 Shaw Prize with Simon Donaldson. Taubes' residence at MSRI this year was made possible by support from the Clay Mathematics Institute and the Simons Foundation.



David Eisenbud

Most postdocs and several participants, including senior research professors such as Denis Auroux, Ko Honda, Richard Montgomery and Cliff Taubes, are staying at the MSRI for the whole academic year. Several other members will come either for one of the semesters, or for several months in different periods of the program. For instance, Alexander Givental, Kai Cieliebak and Octav Cornea are three other research professors in residence during the Fall semester, while Tudor Ratiu split his time between the two semesters. In the Fall semester two of the organizers, Eleny Ionel and Yasha Eliashberg, are in residence.

There are strong ties to the two concurrent semester-long programs that MSRI is running this year: the program on Tropical Geometry (see page 1) and the spring program on Homology Theories of Knots and Links. Both these programs have joint seminars with the symplectic program postdocs and members. It is clear that all these programs greatly benefit from each other. For instance, in the Fall there is a joint seminar between the tropical and symplectic program, and several participants of both programs are actively discussing potential applications to one area of methods and tools developed in the other. Some of the participants, such as M. Abouzaid, D. Auroux, and B. Parker, are actively working in both areas.

In order to make more room for the work and discussions, all weekly seminars of the program (and there are many of them!) are concentrated in two days. Besides the joint seminar with the tropical geometry program, already mentioned, there are the general seminar on symplectic and contact geometry and topology, the Broken Dreams seminar, and several weekly working groups. The Broken Dreams seminar was initiated by Cliff Taubes: participants are encouraged to talk about their favorite projects which so far have not worked out.

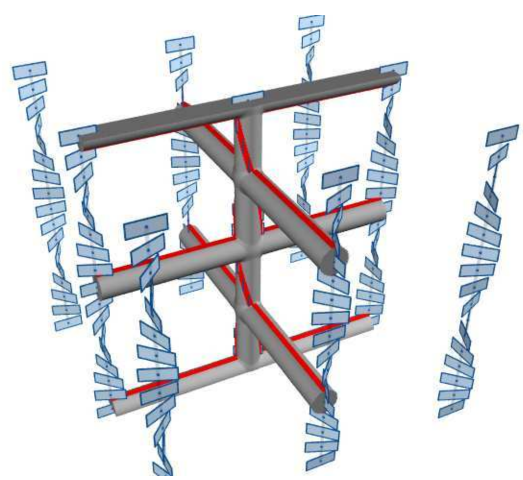
There are four current working groups. Each of the working groups set a goal of completing a certain very particular project. The working group on polyfolds is organized by Oliver Fabert and Joel Fish. Hofer–Wysocki–Zehnder's polyfold theory is a gigantic project of these authors, which aims at the creation of firm analytic foundations for the unified approach to Floer homology, Gromov–Witten theory, symplectic field theory and other algebraic formalisms arising from the study of the topology of moduli spaces of holomorphic curves and other related objects. The goal of the working group is to make a "user guide" for polyfold theory which would be very useful for all mathematicians working in this area.

Dmitri Zvonkine and Oliver Fabert have organized a group with the goal of understanding the connection between integrable systems and symplectic field theory. They began with a series of lectures on integrable systems in Gromov–Witten theory and the Dubrovin theory of Frobenius manifolds and bihamiltonian systems.

Vera Vertesi has initiated a group with the goal of better understanding Giroux's correspondence between contact structures and open books. Many proofs in this field are missing from the literature, and an ambitious goal of the group is to reconstruct them.

Sheel Ganatra, Maksim Maydanskiy and Yakov Eliashberg organized a group on algebraic structures in the theory of holomorphic curves. One of its goals is to prepare the participants of the program for two November workshops, one at MSRI and another at the American Institute of Mathematics in Palo Alto, devoted to this subject. A more particular goal of the group is to understand the relation between Seidel's approach to symplectic topology of symplectic Lefschetz fibrations in terms of the Fukaya category generated by the vanishing cycles, and the Bourgeois–Ekholm–Eliashberg Legendrian surgery formalism. The group seems to be close to reaching this goal. In particular, Ganatra and Maydanskiy have succeeded in deducing Seidel's conjecture about symplectic homology of Lefschetz fibrations of Liouville manifolds from the result of Bourgeois–Ekholm–Eliashberg.

Some of these themes will continue into the second semester of the program, but there will also be fresh areas of concentration. A new subset of organizers — John Etnyre, Dusa McDuff and Paul Seidel — will be in residence. New research professors, such as Emmanuel Giroux, Kenji Fukaya, Victor Ginzburg, Ivan Smith, Leonid Polterovich and Yongbin Ruan will arrive. There will be two more workshops: "Symplectic and Contact Topology and Dynamics: Puzzles and Horizons" and "Symplectic Geometry, Noncommutative Geometry and Physics", the latter of which is cosponsored by the Hayashibara foundation.



Left: Cooling of a liquid crystal into solid crystalline phase. Photograph by Brian Johnstone. The conservative part of the liquid crystal equations are Hamiltonian in the Lie–Poisson structure associated with a semidirect product group with cocycles. Right: Construction of an open book decomposition for the standard contact structure on the 3-torus. Graphics by Patrick Massot.

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