Spring/Fall 2003

EMISSARY

Mathematical Sciences Research Institute

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Notes from the Director

David Eisenbud

One of the pleasures of any University is the renewal that takes place every fall — all those bright, expectant youngsters. With MSRI's ever-changing scientific horizons we have the same thing, but more so; the analogue might be a University where even the faculty changes! Of course some stability is necessary — that's the function of our Advisory Committees and Directorate. The model has worked well. As we go into our twenty-first fall, I feel the pleasure and excitement once again. Our new programs are *Differential Geometry* and *Discrete and Computational Geometry* — matters classical and modern, "pure" and "applied" intimately mixed. We have a mix of interdisciplinary workshops coming up too, with *Genetics of Complex Diseases* and *Mathematical Neuroscience* in the Spring. As usual, you can find more information on our web site.

I returned last July from my semi-sabbatical year. I had lots of fun being a member at MSRI, taking part in the Commutative Algebra year, while involved in fundraising for the building extension (see on the right). Michael Singer held the fort as Acting Director, with Bob Megginson as Deputy. Many people told me that I wouldn't be able to enjoy a sabbatical unless I left town, and apparently they also told Michael that he would have a hard time with me hanging around in the house. I'm happy to report that it wasn't so — Michael and Bob managed things wonderfully, and protected my time for research; and (Michael tells me, anyway) he never felt that I encroached on his turf.

As has often happened to me at the end of a vacation, I felt a happy anticipation about returning to the job of director when the time came. Because I'll be somewhat distracted from MSRI this year by being AMS President, and because there are some big new projects afoot, the Trustees decided to increase the directorate a little, and MSRI will have the privilege of two enormously capable Deputies this year: Bob Megginson is continuing, and Hugo Rossi will join us, returning to MSRI after four years back in Utah.

So much for the cast of characters; let me tell you about some things I find exciting and new in what's going on.

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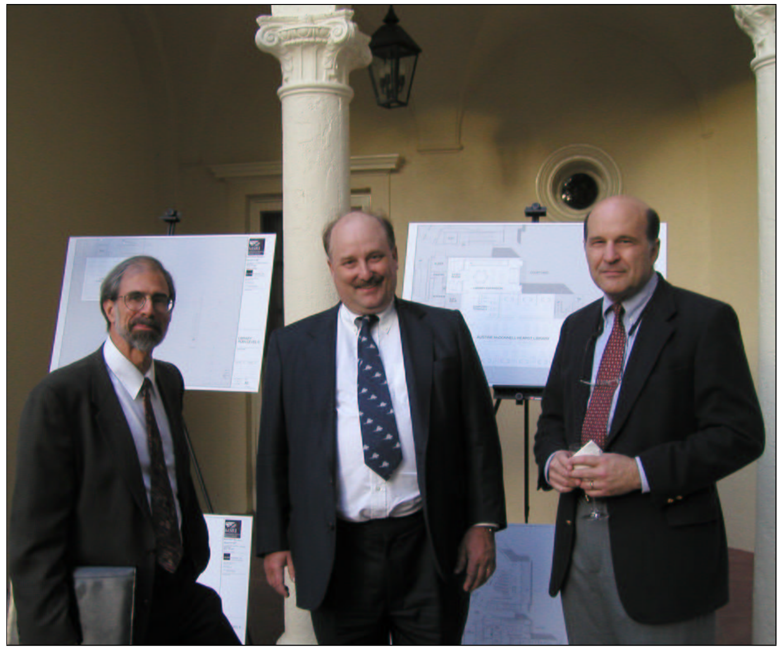

David Eisenbud, William R. Hearst and project architect Bill Glass

Building Campaign Predicts Success

Three significant commitments from individuals to MSRI's $7.3 million campaign will help the Institute announce Campaign success soon. In addition to a generous early grant from founding director Shiing-Shen Chern, major grants have also been received from MSRI trustee and campaign co-chair James H. Simons and from MSRI trustee William R. Hearst. Mr. Simons' generosity will be recognized by the naming of the Simons Auditorium. Mr. Hearst's major commitment will be recognized by the naming of the Institute library for his mother, Austine McDonnell Hearst. Professor Chern's gift and his crucial role as founder of the Institute will be recognized by designating the entire expanded and renovated Institute building Chern Hall.

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Notes from the Director

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Our *building campaign* is going extremely well. We have firm plans to break ground in May 2004, and should be occupying the new space by September 2005. Many mathematicians have contributed the thousand dollars necessary to have a named brick in our new garden (where, of course, there'll be an outdoor blackboard). It's not too late to join their ranks! — go to www.msri.org/development/index.html if you're interested. See page 1 for the announcement of some larger gifts.

We had two very lively *summer graduate programs*, one of them in Portland, Oregon. One reason it's fun to be MSRI's director is that there is such a strong expression of interest and approval of MSRI from the mathematical community. One of the concrete ways this is registered is through the Academic Sponsors organization. There are now 72 Academic Sponsors. One of the main direct membership benefits is to be able to send two graduate students (or three, if at least one is female or minority) to our summer graduate programs. These two-week events can be extremely exciting and motivating, and are one reason why so many departments are able to get help with the cost of being an academic sponsor ($3000/year) from their Deans of Graduate Studies as well as Deans of Science. Next year's lineup of programs and main organizers is:

- Hyperplane Arrangements (Sergey Yuzvinsky)
- Knot Theory and Three-manifolds (Dale Rolfsen)
- Analysis of Algorithms (Gadiel Seroussi)

If your university is not yet in the ranks of our Academic Sponsors, it's not too late to change that and take advantage of these programs.

Many things are happening on the *education* front! MSRI can't fix the troubles that everyone sees in mathematics education, but there are things we can do that will have a positive, perhaps important effect. Front-line mathematical research will always be MSRI's first priority; but the strength of this research depends on enough talented kids learning that mathematics exists and can be fascinating and fun. Much depends, too, on all children learning good mathematical skills and learning to think mathematically.

So one new MSRI initiative this year is to begin the organization of a series of high-level conferences on crucial education issues, where scientific problems mix with problems of policy and reform. To this end, we have formed an Education Advisory Committee, a remarkable group of educators:

- Bruce Alberts (President, National Academy of Sciences, Chair of the National Research Council)
- Michele Artigue (Professor of Mathematics, Université Paris, Vice-President of the International Commission on Mathematical Instruction)
- Deborah Loewenberg Ball, EAC Chair (Professor, School of Education, University of Michigan, former elementary school teacher, and MSRI Trustee)
- Hyman Bass (Professor of Education and Mathematics, University of Michigan, past President of the American Mathematical Society)
- Jim Lewis (Professor and former Chair, Dept. of Mathematics and Statistics, University of Nebraska-Lincoln, 1998 recipient of a Presidential Award for Excellence in Science, Mathematics and Engineering Mentoring)
- Robert Moses (Educator and Civil Rights Activist, Founder and President, The Algebra Project Inc.)
- Alan H. Schoenfeld (Professor of Education, UC Berkeley, Vice President of the National Academy of Education, and a Fellow of the American Association for the Advancement of Science)
- Lee Shulman (President, The Carnegie Foundation for the Advancement of Teaching, Fellow of The American Academy of Arts and Sciences)

One of the first of these Education Conferences will deal with the opportunities and costs in the assessment of students' mathematical knowledge. Plenty of dollars are spent on standardized testing today, but real information on what's being learned is surprisingly subtle and hard to get. There's no question that testing influences what is taught, for better or worse. It also makes change and reform hard — one reason why some favor testing and others oppose it! MSRI's conference will bring together mathematicians, teachers, Education School Faculty and policy makers to discuss the issues and perhaps to agree on elements of a white paper and research agenda in this area.

Of course, we continue our involvement with the Bay Area Math Olympiad (BAMO) and Math Circles (BAMC). These initiatives, founded by Zvezdelina Stankova (an MSRI postdoc at the time), Paul Zeitz (UCSF) and Hugo Rossi (then and now MSRI Deputy Director) have brought kids in grades 7–12 and their teachers together for Sunday sessions through the year with University mathematicians; once a year a three-hour olympiad with five essay-proof questions (very different from the thin multiple choice exams that form the lower levels of the USA Math Olympiad) brings in nearly 300 kids from junior high and high schools, rich and poor, around the Bay Area.

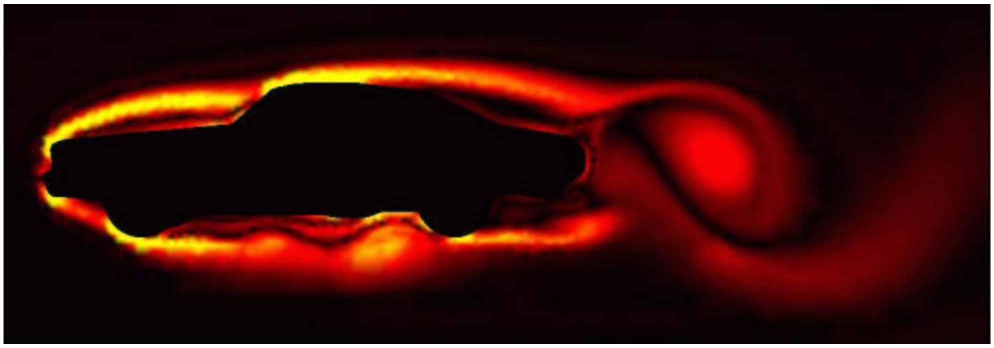
BAMO and BAMC are now in their fifth year, and have been models for similar programs in San Jose, Palo Alto, Utah and other places. What's different in the picture is that we've been able to begin an endowment for this program with a generous grant from the Clowes Foundation; and we've obtained new support, on a scale that will allow more systematic outreach to underprivileged kids and make these proven programs much more successful and stable, from the Hilde Mosse Foundation.

We hope to leverage this support, in the years to come, to help this Mathematical Circles movement become much more widespread. Wish there were a Circle near you? Consider joining with others to start one! We're planning a meeting to help and coordinate those who are doing it, and those who'd like to start. We'll announce a date as soon as possible.

Heavy Metal

Josef Sifuentes – *Rice University '04*

Soon after my freshman year, Dr. Richard Tapia approached me with a challenge. He said he had this car, a 1970 Chevrolet Chevelle Super Sport, that he took to car shows. He wanted a video to accompany it that would encompass the theme "Heavy Metal" and the car amongst an array of fluid flow effects, explosions into particle arrays, morphing guitars and muscle cars. At the time I was working in Rice University's Mechanical Engineering Department on a perfectly rational and sensible engineering project, conducting simulated experiments in computational fluid flow, using a Navier–Stokes solver called XNS, created by Dr. Marek Behr. I told Dr. Tapia that I had no idea how to go about creating the kind of wild images he had described to me.



Our project in engineering was really fascinating. There has always been a certain aesthetic that we find in the way a river breaks apart and flows past a jutting rock, creating rivulets and little eddies; the way we visualize the wind swirling through tree branches. We see evidence of this fascination in van Gogh's *Starry Night* or in the repeated motif of a leaf that follows the currents of a windy day in swirls and glides. There is a certain abstract and natural quality, but at the same time we recognize there are certain rules that this phenomenon follows. These rules are the Navier–Stokes partial differential equations.

With this in mind, I agreed to take on Dr. Tapia's project. I had a chance to assume the role of Mother Nature in my own virtual world, to create fluid flows according to my whims and my idea of the theme "Heavy Metal". I sought an artwork that was visually exciting for car show judges and intellectually interesting to engineers and mathematicians.

But playing nature by manipulating physical and mathematical constraints is a difficult task. Just getting started with the animating process blocked me. I didn't really let loose or feel comfortable with the project until Dr. Tapia told me: "it is ok to fail, everybody fails sometime, that's how you push boundaries. If you work on this, and it ends up in a flaming failure, that's ok, because you as a student learned something and you got paid." This freed me to let loose and follow my passion for artwork, experimentation and mathematics. Failing repeatedly at first, I finally began to create simulations perfectly suited for "Heavy Metal".

Richard Tapia of Rice University has the remarkable ability to figure out what it is that excites his students, and then show them why this should also lead to an excitement about mathematics. A case in point is Josef Sifuentes, an undergraduate at Rice in mechanical engineering and art, whose interest in music videos and muscle cars led to a project suggested by Tapia that found an unusual setting for an application of a finite element solver for the incompressible Navier–Stokes equations. This is Josef's description of the project that he presented, along with the visually and sonically stunning video that resulted from it, at the Blackwell–Tapia Conference held at MSRI in November 2002 (see page 1 in the Fall 2002 Emissary).

XNS is a finite element solver of the incompressible Navier–Stokes partial differential equations. It creates a set of dense nodes as vertices of a triangular pattern mesh that represents the fluid domain. The value within each triangle or element is determined by values at the nodes. The process is computationally expensive, usually requiring hours of CRAY time. I had to budget not only time but also method. My time steps were long so as to cover as much time and action as possible without making the simulation too expensive. This limited my simulations to 2 dimensions, which actually worked to my advantage as it led to a moving poster effect of the video, which is perfect as a car display accessory.

To start the project, I situated the car on a road going at a high Reynolds number (a dimensionless ratio of diameter times velocity over viscosity that governs turbulence, friction, vortex shedding, and things of the sort). I was going for a scene of turbulent flow, hoping that the rear of the car would shed vortices highlighted in reds and oranges and yellows to mimic the flames painted on many hot rods and muscle cars. What I found out was that the car was well engineered. The fluid flowed right over the car and settled smoothly in the back. A success for an engineer, but a boring result for an artist. So in order to create the dynamic scene that I wanted, I lifted the car in midair, and experimented with different Reynolds numbers. Too low a Reynolds's number causes too smooth a flow and prohibits vortex shedding to take place. But with a too high Reynolds number, the frequency of vortex shedding gets so high that the effect is lost in the numerical time step of the solver, so my solutions either settle at a standstill or flash through non-smooth solutions. Vortex shedding frequency is dominated mostly by object diameter, velocity of the fluid, and the Strouhal number, which doesn't vary too much. I settled on a Reynolds number of 5000 to 10,000 with low viscosity. I got very clear and definite vortex shedding that was slow enough for my time step to capture. Using a graphics package to highlight levels of high vorticity with red, mids in yellows and oranges, and none in black, I created a sharp contrast of beautiful molten flames licking past a speeding Chevelle against a black backdrop apparently in mid air. The colors of the flames against the black reinforced the Heavy Metal theme and title, and echo the candy red racing stripes against the black paint on the actual Chevelle body.

As I worked more on the video, I became more interested in the mathematics and the physics of the simulations, leaving behind my engineering thoughts of physically sound simulation criteria. What if the car itself was not the object, but rather the fiery fluid itself, flowing against itself and the irregular outlines of the car's sleek profile? As if the car became red hot and then came to life as a bubbling pot of lava. I mapped the profile of the car to the boundary conditions, with the initial condition on the bumper of very low velocity. This created a certain lava-lamp effect that matched very well with the psychedelic sounds of Led Zeppelin's Trampled Underfoot, which is the soundtrack to the video. I blocked off certain flows at boundaries and created unrealistic pressures and flows. I became totally immersed in what I could create by manipulating the partial differential equations and the boundary and initial conditions, and lost sight of maintaining fidelity to accurate solutions. In fact, many of the simulations showed that the solver did not converge; but the visual feedback of the output was stunning.

Then I became interested in the effect of not only manipulating the boundary and initial conditions, but also the finite element mesh of the program. I had finally left reality behind and became purely interested in how mathematics affects the overall appearance of the moving canvas. Though I was no longer interested in physical accuracy as in the previous simulations, they still maintained the appearance of naturally occurring fluid flow in a given environment enhanced by color emphasis. Generally speaking, a denser mesh produced more accurate physical descriptions of the created simulation. A sparse mesh produces nearly linear results, losing smoothness and any semblance of a relation with naturally occurring phenomena. I wanted to contrast the two effects of smooth flow compared to choppy linear effects that appear very abstract and unrestrained. The simulation was a rectangular pool of fluid that was broken up by the block words, "HEAVY METAL", as carved blocks in a wade pool. Physically it was perfectly plausible. In the mesh of the pool I incorporated the car's shape into a densely populated area of mesh, while the rest of the virtual pool was represented by a very sparse layout of nodes. Physically, it all represented the same empty space. The accuracy and smoothness of the solution became the car. The car was defined by an area of a smooth solution amidst a rushing pool of fluid-like blocks that glowed and flickered in abstraction. This is my favorite effect. The visual appearance is something like watching a Magic-Eye picture in motion but with an image that is immediately obvious.

The video is garnished with other scenes that tie in with the mood that a 1970 muscle car amidst flowing flames would create. In one sequence, a car spins in three dimensions as if from a scene in The Matrix, juxtaposed by the throwback theme and moving poster quality of the video. At one point the words of a car ad featuring a 1970 Chevelle reads: "In ten seconds your resistance will self-destruct" catches fire and ignites a run-through of preceding images that accelerates while this 1970 Chevy Chevelle echoes the furious motion until the images become a blur and the entire scene explodes into the fiery "magic-eye" effect.

In the end what I created was an artwork of found materials. I have delved deep into each of these materials creating an artwork that is visually pleasing, but also alludes to the themes of rebellion, speed, and heavy metal that was deeply intertwined with the nation's attitude in that decade. I benefited from my own exploration into manipulating the Navier–Stokes equations and its solver. But more importantly, perhaps I can impact a future generation. I plan on showing my video as a panelist speaker at the annual Hispanic Career and Education day that draws thousands of K–12 youngsters with wide eyes peering into the future. If a younger and often left-out generation can see that mathematics can play a role in the beautiful things in life, the things that interest them and spark their imagination, such as speeding hot rods, animated flames and visual graphic effects, then they can be convinced that math is not only important, but beautiful. They are impacted and in turn they become our future generation that impacts the world of mathematics and computational sciences in their own way.

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Inside Out: Inverse Problems

Gunther Uhlmann

In 1982–1983, the first year of full operation of MSRI, I had the great fortune of participating in the program on Nonlinear Partial Differential Equations. It was a great program with many interesting visitors and participants. I remember vividly one lecture that changed the direction of my research. Bob Kohn gave a beautiful seminar on his joint work with Michael Vogelius [KV] on an inverse problem that became known in the mathematical community as Calderón's problem [C]. The inverse problem is to determine the conductivity of a body by making voltage and current measurements at the boundary of the body. I became fascinated by this problem and started to discuss it with John Sylvester, with whom I was sharing a house at the time. This led to our joint work and a solution for a class of smooth conductivities in dimension three or larger [SU].

In the last twenty years or so then there has been an explosion of results on this inverse problem and many others. A program at MSRI on inverse problems was very much desired at this stage. In August 1998 at a meeting in Oberwolfach I mentioned to Alberto Grünbaum my thought of proposing a program at MSRI. He encouraged me to submit a proposal with several colleagues, and helped the program become a reality. We jointly organized the workshop on "Mathematics of Imaging" (http://www.msri.org/activities/events/9900/imaging/) that was the successful kick-off of the inverse problems semester.

It is impossible to describe in a few paragraphs all the topics discussed during the MSRI semester. See [U] for survey papers of some of the recent developments. Inverse problems arise in practical situations such as medical imaging, geophysical exploration, and non-destructive evaluation where measurements made on the exterior of a body are used to determine properties of the inaccessible interior. The modeling and development of reconstruction algorithms leads to many basic research questions. I personally find fascinating the continuous interplay between pure and applied mathematics in the study of inverse problems.

This interplay is very striking particularly in the applications of Microlocal Analysis (MA) to inverse problems. MA, which is roughly speaking local analysis in phase space, was developed about 30 years ago by Maslov, Hörmander, Sato and many others in order to understand the propagation of singularities of solutions of partial differential equations. The early roots of MA are in the theory of geometrical optics. For a comprehensive treatise on MA see [H].

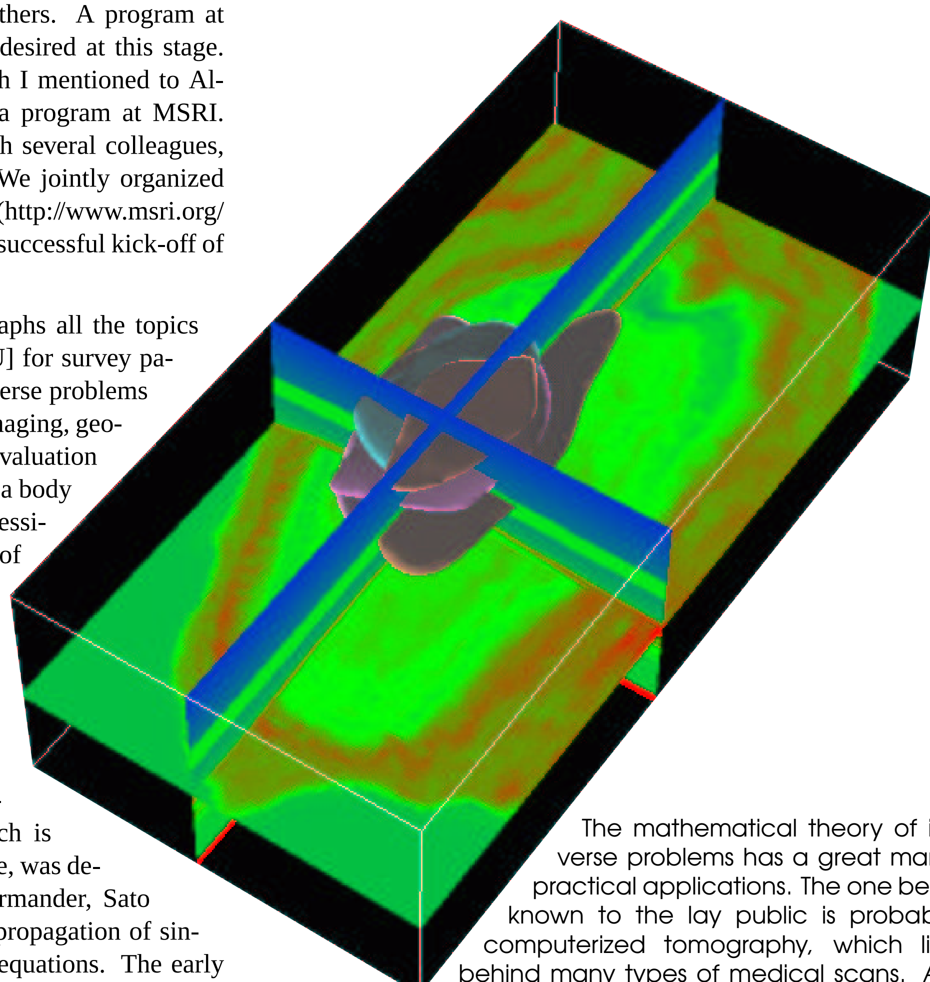
In inverse problems microlocal analysis is helpful in locating the places where a medium has discontinuities, and also in determining the strength of these discontinuities. For several of the applications to inverse problems one can think of this distribution as a function which is smooth away from a collection of disjoint surfaces and has jump discontinuities across the surfaces. A basic concept in MA is the notion of wave front set of a distributions which, roughly speaking, determines the location of the singularities and also the direction of the singularity. Therefore the analysis of singularities is done in phase space. In order to define the concept of wave front set more precisely we need some notation and definitions.

We denote by $C_0^\infty(\mathbb{R}^n)$ the space of smooth and compactly supported functions in Euclidean space. $\mathcal{D}'(\mathbb{R}^n)$ denotes the set of continuous linear functionals on $C_0^\infty(\mathbb{R}^n)$. A distribution u has compact support if there exists a compact set $K \subset \mathbb{R}^n$ such that $u(\phi) = 0$ for all $\phi \in C_0^\infty(\mathbb{R}^n)$ which are zero on K. We denote by $\mathcal{E}'(\mathbb{R}^n)$ the set of compactly supported distributions which can be thought also as the dual space of the space of smooth functions.

The Fourier transform of a compactly supported function f is defined by

$$\widehat{f}(\xi) = \int e^{-i\langle x,\xi\rangle} f(x)\,dx.$$

For a compactly supported distribution the Fourier transform is the



The mathematical theory of inverse problems has a great many practical applications. The one best-known to the lay public is probably computerized tomography, which lies behind many types of medical scans. An equally important area of applications is seismology. The figure, taken from Hoop's article in [U], shows an application of the generalized Radon transform to multi-component ocean bottom seismic data, acquired over the Valhall field in the Norwegian sector of the North Sea, where (it is believed) the presence of gas in the overburden creates lenses that cause caustics to form.

smooth function defined by $\widehat{u}(\xi) = u(e^{-i\langle\cdot,\xi\rangle})$. Here $\langle x, \xi\rangle = \sum_{i=1}^{n} x_i\xi_i$ denotes the scalar product. If we consider the Laplacian in $\mathbb{R}^2$, defined by $\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$, we have

$$\widehat{(-\Delta)f}(\xi) = |\xi|^2\widehat{f}(\xi).$$

This allows to define the square root of the negative Laplacian in $\mathbb{R}^2$ by $(-\Delta)^{1/2}$ which is given by

$$(\widehat{-\Delta^{1/2}})f = |\xi|\widehat{f}(\xi),$$

or, in terms of convolution,

$$(-\Delta)^{1/2}f(x) = \int \frac{f(y)}{|x-y|}dy.$$

A compactly supported distribution is smooth if and only if the Fourier transform decays rapidly in all directions: for any positive integer N, there exists a constant $C_N > 0$ such that

$$|\widehat{u}(\xi)| \leq C_N(1+|\xi|)^{-N}.$$

This relates the smoothness of a distribution to decay of its Fourier transform. The wave front set of a distribution u measures not only the places where the distribution is not smooth, but also the directions in which its Fourier transform does not decay rapidly.

Definition. Let $u \in \mathcal{D}'(\mathbb{R}^n)$ and let $(x_0, \xi_0) \in \mathbb{R}^n \times \mathbb{R}^n\backslash 0$. We say that $(x_0, \xi_0) \notin WFu$, the wave front set of u, if and only if for every $\phi \in C_0^\infty(\mathbb{R}^n)$ with $\phi(x_0) \neq 0$, and for every nonnegative integer N, there exists $C_N > 0$ such that

$$|\widehat{\phi u}(\xi)| \leq C_N(1+|\xi|)^{-N}$$

in a conic neighborhood of ξ_0.

Examples

(a) Let δ_0 be the Dirac distribution concentrated at $0 \in \mathbb{R}^n$, that is, $\delta_0(\phi) = \phi(0), \phi \in C_0^\infty(\mathbb{R}^n)$. Since $\widehat{\delta_0}(\xi) = 1$ does not decay in any direction one can easily show that

$$WF(\delta_0) = \{(0,\xi) \in \mathbb{R}^n \times \mathbb{R}^n\backslash 0; \xi \neq 0\}.$$

(b) Consider in $\mathbb{R}^2$ the Dirac distribution of the plane $H = \{x_1 = 0\}$ which is defined by

$$\delta_H(\phi) = \int \phi(0, x_2)dx_2, \phi \in C_0^\infty(\mathbb{R}^2).$$

The wave front set of this distribution consists of the set of normals to the plane H:

$$WF\delta_H = \{(x,\xi) \in \mathbb{R}^2 \times \mathbb{R}^2\backslash 0; x \in H, \langle\xi, x\rangle = 0 \text{ for all } x \in H.$$

(c) A similar example relevant to inverse problems is the characteristic function of a domain Ω with smooth boundary. We have that $WF\chi_\Omega$ is the set of normals to the boundary ∂D.

We describe a simple application of MA to *computed tomography* (CT), which is probably the inverse problem best known to the general public. In this imaging method the attenuation in intensity of an X-ray beam is measured, and the information from many X-rays from different sources is assembled and analyzed on a computer. Mathematically it is a problem of recovering a function from the set of its line integrals (or the set of its plane integrals). Radon found in the early part of the 20th century a formula to recover a function from this information. The application to diagnostic radiology did not happen until the late 60's with the aid of the increasing calculating power of the computer. In 1970 the first computer tomograph that could be used in clinical work was developed by G. N. Hounsfield. He and Allan M. Cormack, who independently proposed some of the algorithms, were jointly awarded the 1979 Nobel prize in medicine.

We describe now Radon's inversion formula in two dimensions. Let ω be a unit vector. Then the Radon transform of a compactly supported function in $\mathbb{R}^2$ is defined by integrating a function along lines with normal direction ω,

$$Rf(p,\omega) = \int_{\langle x,\omega\rangle = p} f(x)\, ds,$$

where ds denotes the standard Lebesgue measure on the line. The dual transform of the Radon transform, R^t, is the integration of a function for all lines going through a given point:

$$R^t g(x) = \int_{S^1} g(\langle x,\omega\rangle, \omega)\, d\omega,$$

where $d\omega$ denotes the standard measure on the circle.

The Radon inversion formula is

$$f(x) = \frac{1}{2\pi^2}(-\Delta)^{1/2}R^tRf(x)$$

for all compactly supported functions (and distributions).

Notice that the Radon inversion formula is not local, that is in order to determine the function f near a point x_0 we need to know the Radon transform of f for lines far away from x_0. (The non-locality is due to $(-\Delta)^{1/2}$.)

However, since $(-\Delta)^{1/2}(-\Delta)^{1/2}f = -\Delta f$ we derive the formula

$$(-\Delta)^{1/2}f(x) = -R^t\partial_p^2 Rf(x) = \frac{1}{2\pi^2}(-\Delta)R^tRf(x).$$

This formula is local since it involves only differentiating the Radon transform, but it only recovers $\Delta^{1/2}f$ from Rf. However, using the definition above via the Fourier transform one can check that

$$WF(-\Delta)^{1/2}f = WFf.$$

We therefore can recover the singularities of f near x_0 if we know the Radon transform of the function f only for lines passing near to x_0. For more refined applications of this same idea, which is known as *local tomography*, see the survey article of Faridani in [U] and the references given there.

In three dimensions to use X-rays in all possible directions is not practical. Moreover, since the manifold of lines in $\mathbb{R}^3$ is four dimensional, while the object under investigation is a function of three variables, it should suffice to restrict the measurements to a three-dimensional submanifold of lines. A natural case of a three dimensional set of lines are the lines meeting a space curve in $\mathbb{R}^3$: this is motivated by the tomographic scanner design wherein an X-ray source moves on a trajectory in space and for each source point measurements are made on a two-dimensional detector. Local tomography and the study of singularities for this more complicated case is reviewed in [FLU].

In the book [U] there are several other chapters where MA is applied to inverse problems. Maarten de Hoop surveys applications to reflection seismology; Andras Vasy to inverse many-body scattering; Claude Bardos to the analysis of time reversal mirrors; and Vesselin Petkov and Luchezar Stojanov to inverse obstacle scattering. Applications of MA to Calderón's problem is given in [GLU]. A topic of active research in the inverse problems semester was *Synthetic Aperture Radar*(SAR). In this imaging method, an antenna (on a plane or satellite) flies along a path. The antenna emits pulses of electromagnetic radiation in a directed beam perpendicular to the flight path. These waves scatter off the terrain, and the scattered waves are detected with the same antenna. The received signals are then used to produce high-resolution images es of the Earth's terrain. Applications of MA to SAR are given in the papers [CN] and [BC].

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Gunther Uhlmann (gunther@math.washington.edu) is a Professor at the University of Washington, Seattle.

Bouncing Ball Modes and Quantum Chaos

Nicolas Burq and Maciej Zworski

The quantum/classical correspondence is a puzzling issue that has been with us since the advent of quantum mechanics a hundred years ago. Many aspects of it go back to the Newton/Huyghens debate over the wave vs. corpuscular theories of light.

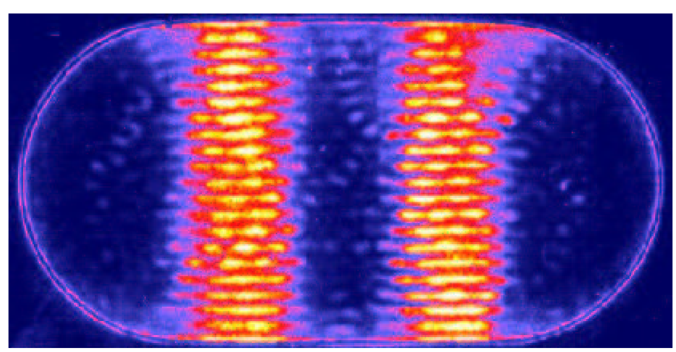
On the surface of our existence we live in a world governed by laws of classical physics. That does not mean that we know precisely how fluids flow or solids move. They are described by highly non-linear rules which are hard to unravel mathematically. Even the simplest classical motion, that of a ball bouncing elastically from confining walls poses many unanswered questions — see http://www.dynamical-systems.org/billiard/ for a fun introduction.

If we investigate deeper, or if we simply use any modern technical device, we come in contact with quantum mechanics. It is governed by a different set of rules which mix wave and matter. The simplest description of a wave comes from solving the Helmholtz equation:

$$(-\Delta - \lambda^2)u = 0\,, \;\; \Delta = \partial_x^2 + \partial_y^2\,, \;\; (x, y) \in \Omega\,, \;\; u|_{\partial\Omega} = 0\,. \quad (1)$$

Here we put our wave inside of a two dimensional region Ω. In classical wave mechanics the limit $\lambda \to \infty$ is described using geometrical optics where the waves propagate along straight lines reflecting in the boundary $\partial\Omega$. Roughly speaking, we expect something similar in the classical/quantum correspondence with the Helmholtz equation replaced by its quantum mechanical version, the Schrödinger equation. For many fascinating illustrations of this we refer to the web art gallery of Rick Heller: http://www.ericjhellergallery.com.

Many researchers on different aspects of semiclassical analysis have been interested in the the correspondence of solutions to the equation above and the classical geometry of balls bouncing from



An experimental image of bouncing ball modes in a Bunimovich stadium cavity. For more details see (CH96) and http://www.bath.ac.uk/~pyscmd/acoustics. With a bit of imagination one can see that the mode has presence in the wings of the table, which is what our theorem below requires.

the walls of Ω: Bäcker, Cvitanović, Eckhardt, Gaspard, Heller, Sridhar, in physics, and Colin de Verdière, Melrose, Sjöstrand, Zelditch, in mathematics, to mention some (see [BS] for references to the physics literature, and [Ze] for mathematics).

Billiard tables for which the motion is chaotic are a particularly interesting model to study. One of the most famous is the Bunimovich billiard table, whose shape is illustrated on the preceding page. By adding two circular "wings" to a rectangular table the motion of a reflecting billiard ball becomes chaotic, or more precisely, hyperbolic, in the sense that changes in initial conditions lead to exponentially large changes in motion as time goes on. (One would probably not want to play billiards on a table like that, and a completely integrable rectangular one can pose enough of a challenge. While discussing billiards and the classical/quantum correspondence we cannot resist mentioning that Pyotr Kapitsa, Physics Nobel Prize 1978, was fond of saying that trying to detect the quantum nature of physical processes at room temperature was like trying to investigate the physical laws governing the collision of billiard balls on a table aboard a ship going through rough seas.)

As a model for studying quantum phenomena in chaotic systems this billiard table has become popular in experimental physics. A genuinely quantum example is in the figure below. It comes from the scanning tunnelling microscope work of Eigler, Crommie, and others.

One question still mysterious to mathematicians and physicists alike is if the states of this system (that is, solutions of the equation above) can concentrate on the highly ustable closed orbits of the classical billiard. *Quantum unique ergodicity* states that there is no such concentration (see [Ze] and references given there). In the arithmetic case, that is for billiards given by arithmetic surfaces where the motion is given by the geodesic flow, spectacular advances have been recently achieved by Bourgain, Lindenstrauss, and Sarnak, while for the popular quantization of the *Arnold cat map* impressive results were produced by Bonechi, De Bièvre, Faure, Nonnenmacher, Kurlberg and Rudnik. Here we describe an elementary but striking result obtained by the authors at MSRI [BZ2].

For a state $u(x, y)$ what matters is its *probability density*, $|u(x, y)|^2\, dx\, dy$ (we assume $|u|^2$ is normalized so its integral over Ω is 1). We say that it is bounded from below in a region if its integral over that region is bounded from below by a positive constant. With this terminology we have, roughly, this result:

Theorem. *For any normalized state of the Bunimovich billiard table, the probability density in any neighbourhood of the wings of the table is bounded from below independently of the energy λ^2 appearing in (1).*

In particular, the result says that single bouncing ball orbits (that is, orbits following an interval perpendicular to the horizontal straight boundaries) cannot produce localized waves. Our result allows concentration on the full invariant set of all vertical orbits over $\mathbb{R}$ — that is consistent with the existing physical literature, both numerical and experimental; see [BS] and [CH96].

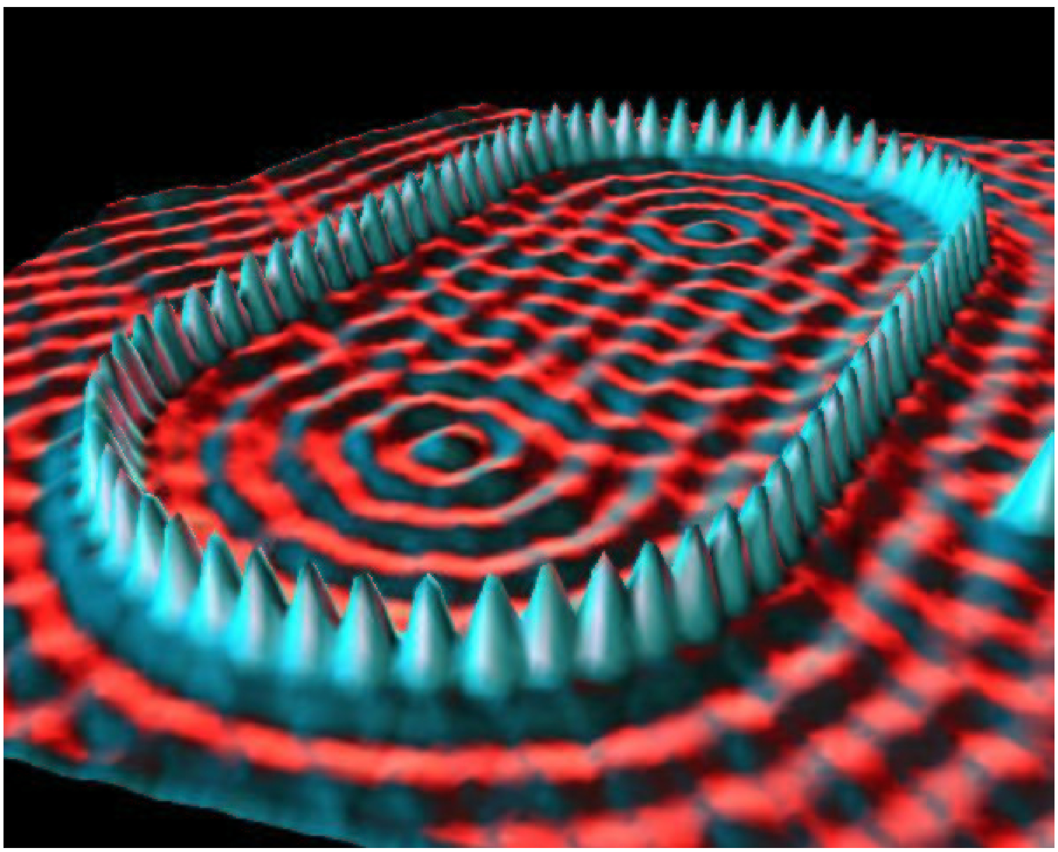
[BS] A. Bäcker, R. Schubert, and P. Stifter. On the number of bouncing ball modes in billiards. *J. Phys. A: Math. Gen.* **30** (1997), 6783–6795.

[BZ2] N. Burq and M. Zworski. Geometric control in the presence of a black box, math.AP/0304184.

[CH96] P.A. Chinnery and V.F. Humphrey. Experimental visualization of acoustic resonances within a stadium-shaped cavity. *Physical Review E* **53** (1996), 272–276.

[Ze] S. Zelditch. Quantum unique ergodicity, math-ph/0301035.

Nicolas Burq (Nicolas.burq@math.u-psud.fr) and Maciej Zworski (zworski@math.berkeley.edu) are professors of mathematics at, respectively, Université Paris Sud, Orsay, France, and University of California, Berkeley. *MSRI held in Spring 2003 a semester on semi-classical analysis, which, in a nutshell, studies the transition between quantum and classical mechanics.*



Quantum corral in the shape of the Bunimovich stadium. Courtesy IBM Research.

Building Campaign

(continued from page 1)

The present library, lecture hall and building will be all but unrecognizable once the work is done. Building Committee Chair Doug Lind, with the close collaboration of MSRI's Director David Eisenbud, and Oakland architect William R. Glass, F.A.I.A., of Glass Associates have created a plan that keeps what is best about the current Institute building (atrium, private offices, space to share ideas), and that improves areas that need improvement.

Several challenge grants have been offered to MSRI's members, alumni, friends and supporters. The $2.5 million Simons Foundation challenge grant will be met with only $200,000 more in contributions to be collected as of this writing. The financial conditions of The Kresge Foundation match have been met, providing $650,000 to the campaign once construction commences.

Groundbreaking for the new building is planned for Spring 2004, and dedication for early 2006.

Simons Auditorium

The Simons Auditorium shape and sloping elevation are carefully designed to accommodate both small groups for working seminars and large lectures in amphitheater style. All chairs will have writing surfaces, and many will face writing tables. The lecture hall's dramatic design and aesthetics will enhance the beauty of the building. New accommodations for audio and video equipment will be state of the art, and will be secure in a separate control booth. With it, the Institute will be able to greatly improve the quality of the streaming video program that has allowed MSRI to broadcast lectures to a global audience over the Internet since 1997.

MSRI trustee and benefactor Jim Simons

Austine McDonnell Hearst Library

Total library area will increase by approximately 150% and shelf capacity by 200%. This will let the library continue to enlarge its choice book collection and archive the almost 100 mathematics journals it subscribes to. The expansion of shelf capacity comes at a time when many libraries are putting paper collections in storage or discontinuing their subscriptions, but MSRI plans to continue offering both paper and electronic documents for maximal convenience. Electronic retrieval of information will be possible from six library computers and from office computers.

The Hearst gift will also make it possible to fund improvements in study space, add to the existing library display space, and establish a dedicated media center where a person or small group can access video material, including recorded Institute lectures, in high-quality MP2 or MP4 format. The central library hall will be a beautiful space with natural lighting and other esthetically appealing elements.

Chern Hall (MSRI Building)

As described in the Fall 2002 Emissary, the new building will have several small rooms for meetings and casual interaction. The Commons — a second floor casual and comfortable space with couches, kitchenette, and an outdoor deck — will be available for socializing. The current first-floor atrium, where visitors are struck by the spectacular view as soon as they walk into the Institute, will be expanded and improved. A break and seminar room will be provided for small meetings, and a boardroom will be available for researchers' meetings and MSRI's governing boards' meetings.

MSRI's outdoor patio, with its popular chalkboard for ongoing mathematical discussions, will be enlarged and partially covered so that it will be useable even in inclement weather.

A new commercial kitchen will make full food service available to researchers and visitors. Improved eating facilities will enhance possibilities for social interaction, and thus mathematical exchange, between researchers.

Storage space is planned on each of the four floors of the new building, which should help avoid unsightly stacks of materials outside offices and in hallways.

Heating, ventilation, and air conditioning needs will be addressed throughout the entire building, and computing infrastructure and capacity will be greatly enhanced.

Academic Sponsors

The following institutions have become MSRI Academic Sponsors since September 2002. For the benefits of academic sponsorship and the list of current Academic Sponsors (now 72), see http://msri.org/governance/sponsors/academicsponsors.html.

University of New South Wales, 11/15/02
University of Nebraska, Lincoln, 1/21/03
Michigan State University, 4/25/03

Puzzles Column

Elwyn R. Berlekamp and Joe P. Buhler

1. One hundred ants are placed on a stick one meter long. Each ant begins to travel either to the left or to the right at a constant speed of one meter per minute. When two ants meet, they bounce off and reverse direction while maintaining their speed. When an ant reaches either end of the stick it falls off.

What is the longest amount of time one must wait to be sure that the stick is completely ant-free?

Comment: We saw this on Ravi Shankar's puzzle mailing at the University of Oklahoma and he found it on Francis Su's *Math Fun Facts* web site at Harvey Mudd College.

2. Let n be a positive integer. (a) Show that some multiple of n has only 0's and 1's in its ordinary base 10 representation. (b) Show that some multiple of 2^n has only 1's and 2's in its base 10 representation.

Comment: This problem will appear in a forthcoming book *Mathematical Puzzles: a Connoisseur's Collection*, by Peter Winkler, published by A K Peters, expected to be available early in 2004. The first puzzle is due to Muthu Muthukrishnan, and the second appeared on the First All Soviet Union Mathematical Competition in 1967.

3. How many disks of diameter 1 fit in a $2 \times N$ rectangle?

Comment: This problem is from *The Inquisitive Problem Solver* (published by the MAA), edited and translated by Paul Vaderlind, Richard Guy, and Loren Larson from a Swedish problem book.

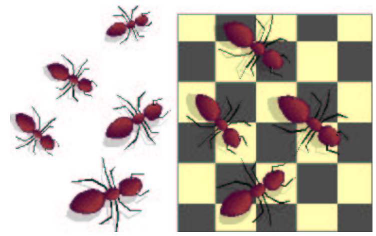
4. A chess game ends on White's sixth move, which is g×f8N, checkmate. (This means that a pawn on KN7 captures a piece on KB8 — the home square of the Black king's bishop — promoting to a knight and checkmating.) Reconstruct the game.

Comment: Richard Stanley and Noam Elkies are working on a book on mathematical aspects of chess, and a preview/teaser appeared in the Winter 2003 *Mathematical Intelligencer*. This is apparently the longest known game in which the last move uniquely determines the game.

5. We have a circular key chain and want to color the keys, using as few colors as possible, so that each key can be identified by the color pattern — that is, by looking at they key's color and neighboring colors as far away as needed. Let $f(n)$ be the minimal number of colors required to uniquely disambiguate a circular key chain of n keys in this way. Determine $f(n)$ for all positive integers n.

For instance, for $n = 4$ the diagram on the right shows that three colors suffice — the keys can be identified as green, blue, red next to green, red next to blue. By checking cases one sees that two colors is impossible (key rings can be flipped over, so references to left, right, clockwise etc. are forbidden). All in all, we conclude that $f(4) = 3$.

Comment: We learned of this problem from Stan Wagon, who included it in his book *Which way did the bicycle go?*; it was originally due to Frank Rubin in the *Journal of Recreational Math*.



Graphic: Mike McGill

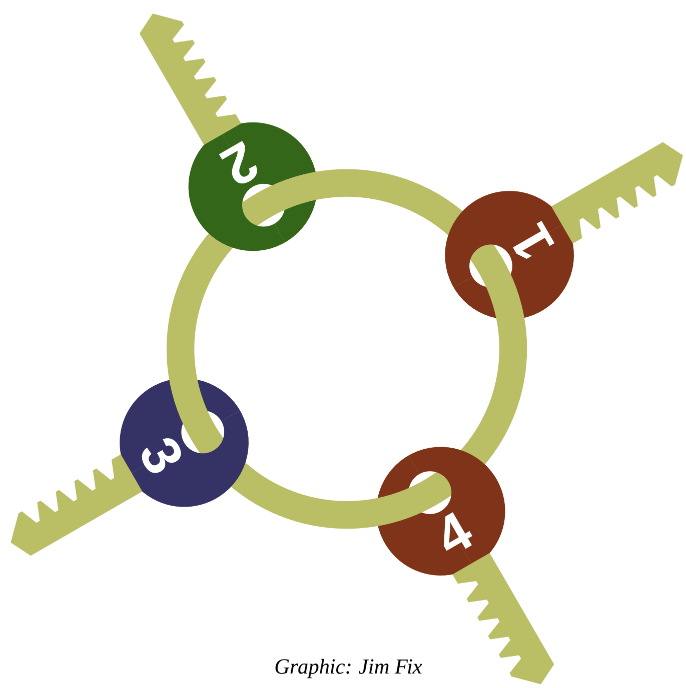
6. A read-only array of length n contains entries $a[0]$, $a[1]$, …, $a[n-1]$ all taken from the set $\{1, \ldots, n-1\}$. By the pigeonhole principle there is a duplicated element (at least one).

(a) Assume that there is exactly one duplicated entry. Find an algorithm (e.g., a program in your favorite programming language) that takes this array as input and then prints out the duplicated value. The program cannot modify the input array. It should run in linear time, and it should use a constant amount of storage words (each capable of storing integers up to n).

(b) Write a program, subject to the same constraints, that prints a duplicated element, without making any assumptions on the number of duplicated entries. This program, too, is forbidden from modifying the input, should run in linear time, and should use constant extra space.

Comment: Part (b) seems to be a lot harder than part (a), and it apparently requires entirely different techniques. We learned of problem (b) from Eric Roberts, who heard it at a SIGCSE meeting in Greece this summer. (Readers aware of earlier sources can share that information by sending email to berlek@math.berkeley.edu or jpb@reed.edu.)

Solutions will be posted in the near future at the Emissary web page, http://www.msri.org/publications/emissary/.



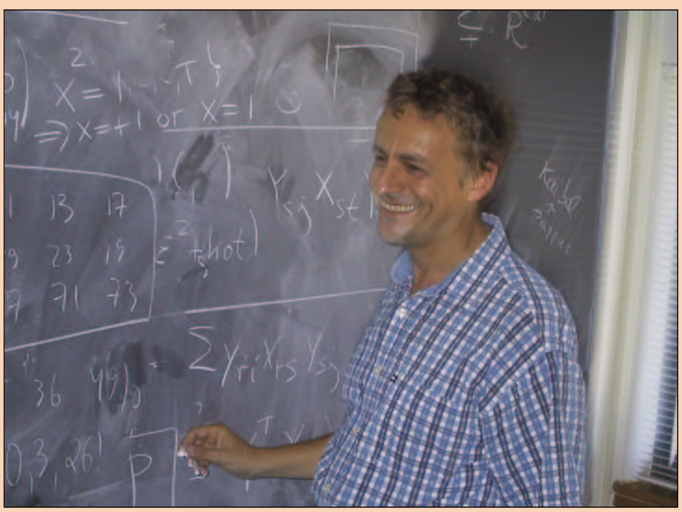
Graphic: Jim Fix

Bernd Sturmfels: Hewlett–Packard Visiting Research Professor, 2003–2004

Professor Bernd Sturmfels of the University of California at Berkeley, and a member at MSRI in the programs on Discrete and Computational Geometry and Topology of Real Algebraic Varieties, is our Hewlett-Packard Visiting Research Professor for the 2003–2004 academic year. Sturmfels has been at UCB since 1995, was a fellow of the Sloan Foundation 1991–1993 and a David and Lucile Packard Fellow in the period 1992–1997. He helped organize the MSRI programs in Symbolic Computation in Geometry and Analysis in Fall 1998 and in Commutative Algebra, 2002–2003.

Since 1999, Hewlett-Packard Laboratories have provided funds to support a Hewlett-Packard Visiting Research Professor at MSRI, selected by a joint MSRI/HP Scientific Committee. This position has been filled by researchers at the top of their fields who come to MSRI to do research and give scientific guidance. The Hewlett-Packard Visiting Research Professor also collaborates with researchers at Hewlett-Packard Labs on problems of mutual interest, often involving graduate students and postdoctoral fellows in these projects. The past holders of this position have been Richard Karp, Hendrik W. Lenstra, Gabor T. Herman, Sergio Verdu and Sandu Popescu.

Sturmfels will work with researchers from HP on problems related to various areas of interest to the Labs, including coding theory, computational geometry, and algebraic aspects of statistical modeling.



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Motivic Integration

Karen E. Smith

Disclaimer: The author is a believer in the white lie as a way to make technical subjects more accessible. A more accurate survey of this topic is the paper of Looijenga: math.AG/0006220.

Motivic integration was introduced by M. Kontsevich in a 1995 lecture in Orsay, where he announced an affirmative solution to the following conjecture of Batyrev: *two birationally equivalent Calabi–Yau manifolds have the same Hodge numbers.* This conjecture was motivated by work in theoretical physics, namely string theory, which predicts that as a manifold the universe is locally a product of $\mathbb{R}^4$ (space-time) and a compact component (a Calabi–Yau manifold). The conjecture means that two such special manifolds share the same important numerical invariants, provided that they contain large enough isomorphic open subsets.

Kontsevich gave a remarkably elegant and conceptual proof of this result, essentially as a corollary of his theory of motivic integration. He was inspired by the theory of p-adic integration, which Batyrev himself had used to prove a weaker form of his conjecture.

Motivic Integration

There is an associated theory of motivic integration for each smooth complex variety. Like any integration theory, we need

1. a space on which the integration theory is defined,
2. a measure on (certain) subsets of that space,
3. some interesting measurable functions, and
4. an understanding of how integrals transform under change of coordinates.

The measures and functions take values not in the real numbers, but in a more exotic ring $\mathcal{M}$, called the *motivic ring.* The elements of $\mathcal{M}$, roughly speaking, are formal $\mathbb{Z}$-linear combinations of varieties with addition corresponding to disjoint union of varieties and multiplication corresponding to direct products of varieties. The ring $\mathcal{M}$ also contains formal multiplicative inverses of certain elements, much in the way the ring of rational numbers is obtained by adding formal multiplicative inverses of integers. This ring is not ordered, so properties of a measure such as "subadditive on unions" do not make sense. However, other basic features of measure theory, such as additivity on finite disjoint unions, do carry over. We discuss each of these four items below.

(1) The arc space of X. Fix a smooth complex variety X. Motivic integration theory for X is an integration theory on the *arc space* $J_\infty(X)$ of X. The points of this space correspond to the "formal arcs" on X. An arc on X can be thought of as an infinitesimal curve centered at a point of X—that is, an arc is a choice of a point, together with a tangent direction at that point, a second order tangent direction, a third order tangent direction and so on. Thinking in this way, it is easy to believe that $J_\infty(X)$ is an infinite dimensional affine bundle over X. Technically speaking, an arc is a map of schemes $\operatorname{Spec}\mathbb{C}[[t]] \to X$.

(2) The measure. Motivic integration theory assigns to certain subsets of the arc space, called *cylinder sets*, a value from the motivic ring $\mathcal{M}$. Roughly speaking, a cylinder set is a set in the infinite dimensional space $J_\infty(X)$ defined by a finite amount of data.

Although the arc space $J_\infty(X)$ is infinite dimensional, it is approximated by finite dimensional varieties $J_m(X)$ called the *jet schemes* of X. The space $J_m(X)$ parameterizes the m-*truncated arcs* on X, where an m-truncated arc is can be thought of the choice of a point together with the choice of an i-th order tangent direction at that point for all $i \leq m$. There is a natural *truncation map*

$$\pi_m : J_\infty(X) \to J_m(X)$$

sending each arc to its truncation at the m-th term.

A cylinder set is a subset of the arc space $J_\infty(X)$ obtained from one of these finite dimensional approximating spaces $J_m(X)$. More precisely, a cylinder set is any subset of $J_\infty(X)$ of the form $\pi_m^{-1}(B_m)$, where B_m is a subvariety of $J_m(X)$. In this case, we can assign to the set $B = \pi_m^{-1}(B_m)$ the measure

$$\mu_X(B) = [B_m] \cdot \mathbb{L}^{-nm},$$

where $[B_m]$ is the class of B_m in $\mathcal{M}$, $\mathbb{L}$ is the class of the affine line in $\mathcal{M}$ (so that $\mathbb{L}^{-1}$ is its formal multipicative inverse) and n is the dimension of X. It is easy to check that this measure is well-defined, using the fact that the natural truncation maps from $J_{m+1}(X)$ to $J_m(X)$ are affine bundles of fiber dimension n.

As a quick example, which is incidentally crucial to the proof of Kontsevich's theorem, let us compute the integral of the constant function 1 over the whole arc space $J_\infty(X)$. Note that $J_\infty(X)$ is the preimage of the "total" truncation map $J_\infty(X) \to J_0(X) = X$. Thus its measure is

$$\mu(J_\infty(X)) = [X] \cdot \mathbb{L}^{n \cdot 0} = [X].$$

Therefore

$$\int_{J_\infty(X)} d\mu_X = [X], \quad \text{the class of } X \text{ in } \mathcal{M}.$$

(3) Some interesting functions to integrate. Now fix any subvariety D of our smooth variety X. The subvariety D gives rise to an integer valued function F_D on the arc space $J_\infty(X)$, given, roughly speaking, by order of tangency of each arc along D. The value of $F_D(\gamma)$ may be zero (when γ misses D completely) or infinite (when γ is tangent to D to every order), or any finite number in between. The definition extends naturally to the case where D is a *divisor* on X, meaning a formal Z-combination of codimension one subvarieties of X. In technical terms, the value of F_D along an arc

$$\gamma^* : \mathrm{Spec}\mathbb{C}[[t]] \to X$$

is the degree of the pullback divisor $\gamma(D)$ on the curve $\mathrm{Spec}\mathbb{C}[[t]]$.

One example of an interesting measurable function on $J_\infty(X)$ is the function $\mathbb{L}^{-F_D}$. Because it takes discrete values, its integral over $J_\infty(X)$ is really just a large sum, which can be shown to converge in $\mathcal{M}$ (in a suitable topology).

(4) Kontsevich's Birational Transformation Rule. The result that gives the theory of motivic integration its power is a formula for how motivic integrals transform under birational change of space. Specifically, let $f : X \to Y$ be a proper birational morphism of smooth algebraic varieties. The Jacobian of this map defines a divisor $K_{X/Y}$ called the *relative canonical divisor* of the map. Algebraic geometers also recognize $K_{X/Y}$ as the unique divisor supported entirely on the exceptional set of f and belonging to the divisor class $K_X - f^*(K_Y)$, where K_X and K_Y denote the divisor classes associated to the canonical bundles on X and Y respectively. In its simplest form, Kontsevich's birational transformation rule states that for any divisor D on Y,

$$\int_{J_\infty(Y)} \mathbb{L}^{-F_D} \, d\mu_Y = \int_{J_\infty(X)} \mathbb{L}^{-F_{f^*(D)+K_{X/Y}}} \, d\mu_X.$$

This theorem is non-trivial to prove, but essentially uses only fairly basic commutative algebra.

The Proof of Kontsevich's Theorem

Having outlined this much of the theory of motivic integration, it is easy to understand Kontsevich's proof of the opening conjecture. First let us first understand the conjecture more precisely.

A Calabi–Yau manifold is a smooth complex projective algebraic variety of dimension n admitting a nowhere vanishing holomorphic n-form — in other words, it has trivial canonical bundle. (The precise definition of a Calabi–Yau manifold also requires certain cohomology groups to vanish, but this assumption is not necessary for Kontsevich's theorem.) Two complex manifolds are birationally equivalent if there are mutually inverse maps between them given locally by rational functions in local coordinates; these maps need not be defined on the closed sets where the denominators of the rational functions vanish. For any non-negative integers p and q, the Hodge number $h_{pq}(X)$ is the dimension of the space of closed (smooth) pq-forms on X modulo the space of exact pq-forms on X; alternatively, $h_{pq}(X)$ is the dimension of the cohomology group $H^p(X, \Omega_X^q)$ where Ω_X^q is the sheaf of holomorphic q-forms on X. Putting this all together, Kontsevich's theorem states:

Theorem. *If X and Y are birationally equivalent smooth complex projective varieties, each of whose canonical bundle is trivial, then $h_{pq}(X) = h_{pq}(Y)$ for all $p, q \geq 0$.*

To prove this, Kontsevich considers a map from the set of all smooth projective varieties to the polynomial ring $Z[u, v]$, defined by sending a variety X to the polynomial

$$\sum_{pq} h_{pq}(X) u^p v^q.$$

His idea is that (using some basic Hodge theory) this map factors through the motivic ring $\mathcal{M}$. The point is then to show that two birationally equivalent Calabi–Yau manifolds map to the same element in $\mathcal{M}$. Therefore, mapping further to $Z[u, v]$, they must of course have the same Hodge numbers.

In factoring the map through $\mathcal{M}$, a variety X is sent to the element it determines in $\mathcal{M}$, or its class $[X]$. As we computed above, the class $[X]$ can be interpreted as the integral

$$\int_{J_\infty(X)} d\mu_X = [X].$$

Noting that the constant function 1 can be written as $\mathbb{L}^{-F_\varnothing}$, where $\varnothing$ is the trivial divisor, we also have

$$\int_{J_\infty(X)} \mathbb{L}^{-F_\varnothing} d\mu_X = [X].$$

Now the point is to use the birational transformation rule to compute this integral in a different way. Let Z be any smooth variety admitting a proper birational morphism $g : Z \to X$ to a variety X. If X has trivial canonical bundle, then

$$K_{Z/X} = K_Z - g^*(K_X) = K_Z,$$

which is independent of X. So using Kontsevich's birational transformation rule, we see that

$$\begin{aligned}[X] = \int_{J_\infty(X)} \mathbb{L}^{-F_\varnothing} d\mu_X &= \int_{J_\infty(Z)} \mathbb{L}^{-F_{g^*(\varnothing)+K_{Z/X}}} d\mu_Z \\ &= \int_{J_\infty(Z)} \mathbb{L}^{-F_{K_Z}} d\mu_Z.\end{aligned}$$

The remarkable thing to notice about this computation is that $[X]$ depends only on Z. But, if X and Y are birationally equivalent, there always exists a smooth variety Z admitting a proper birational morphism to both X and Y (a so-called *resolution of indeterminacies*). So in this case, if both X and Y have trivial canonical bundles, then the classes $[X]$ and $[Y]$ in $\mathcal{M}$ can both be computed as the same motivic integral on Z. This means that X and Y have the same image in $\mathcal{M}$, and hence the same image in $Z[u, v]$ and the same Hodge numbers.

Other Applications of Motivic Integration

This beautiful proof of Kontsevich is only the beginning of a long story featuring many elegant applications of motivic integration to problems in algebraic and arithmetic geometry. For one thing, Denef and Loeser have developed the theory for singular varieties X, as many applications require. In our seminar, we focussed on applications to higher dimensional birational geometry, a program that started with the 2001 Berkeley PhD thesis of Mircea Mustata. For example, using motivic integration, Mustata shows that a singular hypersurface X has *rational singularities* if and only if the jet schemes $J_m(X)$ are all irreducible. Furthermore, he also finds formulas for a subtle invariant called the *log-canonical threshold* of a hypersurface singularity in terms of the dimensions of these jet schemes. Later work of Ein, Mustata and Yasuda establishes a version of the famous *inversion of adjunction* conjecture, which is then applied to prove the following long open conjecture: *no smooth hypersurface of degree* n *in* P^{n+1} *is birationally equivalent to projective space, for* $4 \le n \le 12$.

The elegant proofs of these concrete results are just some of the topics we studied in our Motivic Integration seminar in 2002–2003 at MSRI, as part of the Special Year in Commutative Algebra. Some of our notes and a collection of electronic literature can be found on our seminar web site: www.mabli.org/jet.html.

Karen Smith (kesmith@umich.edu) is a Professor at the University of Michigan, Ann Arbor, MI, 48109-1109.

Work partially supported by the Clay Foundation and by the US National Science Foundation Grant DMS 00-70722.

MSRI Goes Wireless

To cater to the needs of the more than 1500 scientists who, every year, spend anywhere from a week to many months at MSRI, we installed as far back as late 2002 a simple wireless network so that users, using their laptops, may connect to the network from most parts of the building. This is especially important for short-term visitors who don't have an office.

The current configuration uses both a Linksys 2 Access Port and Apple Airports (the latter donated by Chris Heegard). To have wireless access, incoming members and workshop participants need to have a wireless card in their computer (a few cards are available for borrowing), and must also receive an authentication key from computing personnel.

Because some areas, like the lecture hall, don't receive signals well because of interference or blockage, Rachelle Summers (Head of Computing) and Max Bernstein (Network Administrator) are currently improving the wireless system. The upgrade, scheduled to roll out before the end of 2003, will likely involve additional access ports and the installation of antennas, and possibly a switch from the current 802.11b protocol, which tranfers 11 megabits per second, to 802.11g, which tranfers up to 54 Mbits/s.

MSRI seeks to offer its members and visitors the best possible facilities. Fortunately a reliable and capacious wireless system, capable of supporting use even outdoors around the building, is quite affordable nowadays.

Rachelle Summers, MSRI Head of Computing

Bob Osserman Receives 2003 JPBM Communications Award

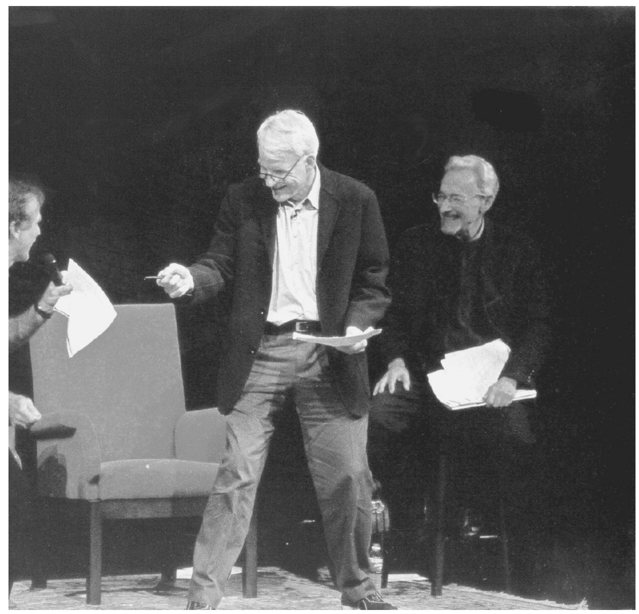
At the annual mathematical Joint Meetings in January at Baltimore, Bob Osserman was presented the 2003 Communications Award of the Joint Policy Board for Mathematics (JPBM). Bob has served on the MSRI Board of Trustees (1979–83), as Deputy Director (1990–1995) and since then as Special Projects Director. Following is an excerpt from the JPBM citation:

For many years, Bob Osserman has been an erudite spokesman for mathematics, communicating its charm and excitement to thousands of people from all walks of life. His slim volume Poetry of the Universe has been described as "artful and beguiling", introducing readers to the inherent beauty and power of mathematical thinking. Bob Osserman believes in making mathematics accessible to the general public. He has done more than explain mathematics, however. He has made mathematics appreciation more than the title of a course: Bob Osserman has changed people's attitudes towards the subject.

Everyone connected with MSRI knows about Bob's work and the tremendous effect he has had through his productions of programs for general audiences. Since the last *Emissary* the most popular event organized by Bob and MSRI (in association with City Arts and Lectures, Inc.), was *Funny Numbers: An Evening with Steve Martin in conversation with Bob Osserman*, which took place on December 15, 2002, at the Herbst theatre in San Francisco.

Steve Martin is renowned for his acting and comedic writing; less known is his interest in the interplay between science and art, as explored in his play *Picasso at the Lapin Agile*, first presented in the fall of 1993 at Chicago's Steppenwolf Theater, which went on to become an audience favorite around the country. The plot involves a fictional meeting between Picasso and Einstein in a Paris cafe at the beginning of their careers, when they are about to stand the art and science worlds on their heads. A film adaptation of the play is scheduled for production in 2003.

At the sold-out event, the audience of almost 1000 was in stitches for much of the time — especially after actor Robin Williams emerged from the audience to join Bob and Steve! The photo below comes from the streaming video of the event. The CD will shortly be available for $25 at www.ams.org, the American Mathematical Society's web site.



Robin Williams, Steve Martin and Bob Osserman extemporize.

Forthcoming Workshops

Most of these workshops are offered under the auspices of one of the current programs (see Director's Notes starting on page 1). For more information about the programs and workshops, see http://www.msri.org/calendar.

January 12 to January 16, 2004: *Introductory Workshop in Topological Aspects of Real Algebraic Geometry*, organized by Selman Akbulut, Grisha Mikhalkin, Victoria Powers, Boris Shapiro, Frank Sottile, and Oleg Viro.

February 9 to February 13, 2004: *Genetics of Complex Disease*, organized by Jun Liu, Mary Sara McPeek, Richard Olshen (chair), David O. Siegmund, and Wing Wong.

February 23 to February 27, 2004: *Topology and Geometry of Real Algebraic Varieties*, organized by Viatcheslav Kharlamov, Boris Shapiro, and Oleg Viro.

March 15 to March 19, 2004: *Mathematical Neuroscience*, organized by Paul C. Bressloff, Jack D. Cowan (chair), G. Bard Ermentrout, Mary Pugh, and Terry J. Sejnowski.

March 22 to March 26, 2004:: *Symplectic Geometry and Mathematical Physics*, organized by Denis Auroux, Dan Freed, Helmut Hofer, Francis Kirwan, and Gang Tian.

April 12 to April 16, 2004: *Algorithmic, Combinatorial and Applicable Real Algebraic Geometry*, organized by Lalo Gonzalez-Vega, Victoria Powers, and Frank Sottile.

June 14 to June 18, 2004: *Analysis of Algorithms*, organized by P. Flajolet, P. Jacquet, H. Prodinger, G. Seroussi, R. Sedgewick, W. Szpankowski, B. Vallée, and M. Weinberger.

June 22 to June 25, 2004: *Tenth Annual Conference for African American Researchers in the Mathematical Sciences*.

Current and Recent Workshops

Most recent first. For information see http://www.msri.org/calendar.

December 1 to December 5, 2003: *Geometric Analysis*, organized by Ben Chow, Peter Li, Richard Schoen (chair), and Richard Wentworth.

November 17 to November 21, 2003: *Combinatorial and Discrete Geometry*, organized by Jesús A. De Loera, Jacob E. Goodman, János Pach and Günter M. Ziegler.

November 9 to November 13, 2003: *Floer homology for 3-manifolds*, organized by Yasha Eliashberg, Robion Kirby and Peter Kronheimer.

October 13 to October 17, 2003: *Mathematical Foundations of Geometric Algorithms*, organized by Pankaj Agarwal, Herbert Edelsbrunner, Micha Sharir, and Emo Welzl.

September 27 to September 28, 2003: *Texas Southern University/MSRI Workshop on Modern Mathematics: An Introduction to 2004–05 Programs at the Mathematical Sciences Research Institute*, organized by Nathaniel Dean and Robert Megginson.

August 20 to August 29, 2003: *Introductory Workshop in Discrete and Computational Geometry*, organized by Jesús A. De Loera, Herbert Edelsbrunner, Jacob E. Goodman, János Pach, Micha Sharir, Emo Welzl, and Günter M. Ziegler.

August 11 to August 20, 2003: *Von Neumann Symposium on Complex Geometry, Calibrations, and Special Holonomy*, organized by Robert Bryant (Co-chair), Simon Donaldson, H. Blaine Lawson, Richard Schoen, and Gang Tian (Co-chair).

June 22 to June 26, 2003: *Preparatory Workshop for the 2003 AMS/MSRI von Neumann Symposium*, organized by Robert Bryant.

May 5 to May 9, 2003: *Mathematical Semi-Classical Analysis*, organized by J. Sjostrand, S. Zelditch, and M. Zworski.

April 21 to April 25, 2003: *The History of Algebra in the Nineteenth and Twentieth Centuries*, organized by Jeremy J. Gray and Karen Hunger Parshall.

April 7 to April 11, 2003: *Semiclassical Methods in Physics and Chemistry*, organized by R. Littlejohn, W. H. Miller, and M. Zworski.

March 29 to April 3, 2003 at the Banff International Research Station: *Commutative Algebra and Geometry*, organized by Mark Green, Juergen Herzog, and Bernd Sturmfels (chair).

March 13 to March 15, 2003: *Computational Commutative Algebra*, organized by Serkan Hosten, Craig Huneke, Bernd Sturmfels (chair), and Irena Swanson.

February 3 to February 7, 2003: *Commutative Algebra: Interactions with Homological Algebra and Representation Theory.*, organized by Luchezar Avramov (chair), Ragnar Buchweitz, and John Greenlees.

MSRI Book Series

Two new books have come out in 2003 so far:

Galois Groups and Fundamental Groups, edited by Leila Schneps, vol. 41

Inside Out: Inverse Problems, edited by Gunther Uhlmann, vol. 47 (see also pages 5–7 of this *Emissary*).

The following titles will appear in the next few months:

Modern Signal Processing, edited by Dan Rockmore and Dennis Healy Jr., vol. 46.

Electromagnetic Theory and Computation: A Topological Approach, by Paul Gross and Robert Kotiuga, vol. 48.

Rankin L-series, edited by Henri Darmon and Shouwu Zhang, vol. 49.

Algorithmic Number Theory, edited by Joe Buhler and Peter Stevenhagen, vol. 44.

Finsler Geometry, edited by D. Bao, R. Bryant, S.-S. Chern, and C. Shen, vol. 50.

Commutative Algebra, edited by Craig Huneke et al., vol. 51.

We will also be printing a second (revised) edition of Calvin Moore and Claude Schochet's *Global Analysis on Foliated Spaces*, which has long been out of print. Back by popular demand!

See http://www.msri.org/publications/books for details on published books and full text for all but the most recent books.

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