Fall 2010

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Mathematical Sciences Research Institute

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MSRI's Core Funding Renewed by NSF

A word from Director Robert Bryant

For the past two years, we at MSRI have been engaged in preparing for and participating in the quinquennial Recompetition for National Science Foundation (NSF) funding through the Mathematical Sciences Research Institutes program (RFP-NSF08565). We submitted our proposal in February 2009, to the Division of Mathematical Sciences (DMS).

The proposal sought a five-year renewal of NSF funding to support the Institute's fundamental mission to advance scientific research and collaboration among mathematicians hosted through MSRI's programs and workshops. The proposal made a strong case for a substantial increase in funding over the level of the past ten years of the NSF Core Grant. It was written by the Directorate in consultation with the Institute's Recompetition Committee and with the invaluable advice of Board members and stakeholders in MSRI's far-reaching scientific community. In August 2009, the NSF sent us the extremely positive panel reviews of our proposal.

As part of the next step in the NSF's Recompetition procedure, a three-day Site Visit was held at MSRI in late September 2009. The NSF Site Visit Team — six DMS staff members plus six outside mathematicians — made arrangements to meet with members of MSRI's Directorate and others closely involved in the governance and operation of MSRI. The Site Visit Team heard from members of the Board of Trustees, the Scientific Advisory Committee, the Human Resources Advisory Committee, and the Educational Advisory Committee, as well as organizers and members of programs that have been held at the Institute and MSRI Postdoctoral Fellows.

MSRI's team, the Directorate and staff, produced a 400-page briefing book of data and documentation that would help the Site Visit Team get a broad overview of the daily workings of the Institute and provide insights into the many ways that the Institute serves

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Anne Brooks Pfister

SF Math Circle Kicks Off Fall Meetings at New Venue with Visit from School District Superintendent

The San Francisco Math Circle (SFMC) began its first meeting at Mission High School by hosting the visit of San Francisco United School District Superintendent Carlos Garcia on September 13, 2010. Superintendent Garcia attended the math circle meeting, observing a room brimming with the energy of 60 middle- and high-school students working collaboratively in groups to solve math problems, and he enthusiastically joined their ranks by accepting his own SFMC T-shirt (left in the photo below, next to Mission High School Principal Eric Guthertz).

The SFMC differs from typical math circles in its large size, and inclusion of teachers, undergrads and graduate students. The 9/13 meeting was run by Paul Zeitz and Brandy Wiegers, the SFMC Director and Associate Director, and Kentaro Iwasaki, an SFMC leader and math teacher at Mission. MSRI Associate Director Dave Auckly was also present.

Anne Brooks Pfister

The SFMC has generous support from the Moody's Foundation and the S. D. Bechtel, Jr. Foundation, besides MSRI.

Random Matrix Theory, Interacting Particle Systems and Integrable Systems

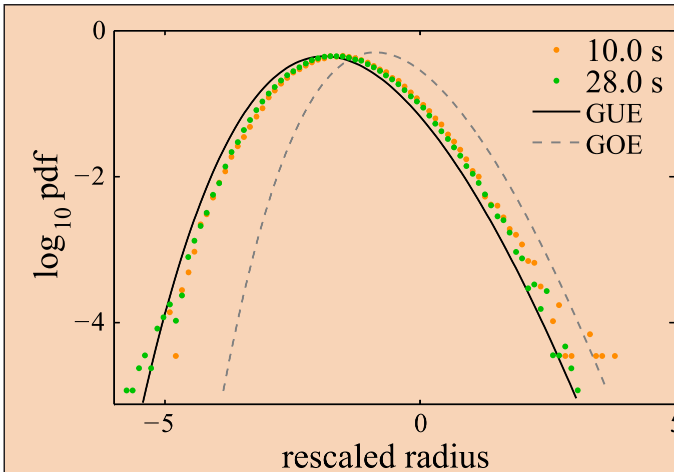
Percy Deift

The current semester program on random matrix theory (RMT), interacting particle systems (IPS) and integrable systems (IS) is a sequel to the highly successful program on RMT and related topics that was held at MSRI in 1999. The late 1990s was a particularly exciting time in RMT: general universality results for unitary ensembles had been established and were fresh off the press, and a fundamental link had been established between Ulam's longest increasing subsequence problem in combinatorics and RMT, particularly the Tracy–Widom distribution for the largest eigenvalue of a matrix from the Gaussian Unitary Ensemble. In the 1950s Wigner had introduced RMT as a model for the scattering resonances of neutrons off a heavy nucleus, and in the 1970s Montgomery had established a remarkable link between the statistics of the zeros of the Riemann zeta function on the critical line, on the one hand, and RMT, on the other. Now, combinatorics and related areas were in the game, and there was much anticipation of developments to come. In particular, there were key conjectures concerning both the internal structure of RMT, such as universality conjectures, as well as applications.

In the decade following 1999, the development of RMT has been explosive and many key conjectures have been settled. Here are some examples, which reflect the work of many authors:

- Universality has been established for orthogonal and symplectic ensembles with very general weights, both in the bulk and at the edge.
- Universality has been established for Wigner and related ensembles, both in the bulk and at the edge. The asymptotic behavior of Toeplitz determinants with Fisher–Hartwig singularities, of the kind that arose in Onsager's solution of the Ising model, have been established in the general case, verifying in particular the conjecture of Basor and Tracy.
- In recent work on random particle systems/random growth models, the Asymmetric Simple Exclusion Process (ASEP) has been shown to exhibit RMT behavior. This result is particularly striking as ASEP lies outside the class of determinantal point processes. Seminal work has also been done on solutions with RMT-characteristics of the KPZ equation, which is believed to provide a universal model for wide classes of random growth processes.
- Free probability theory has emerged as a powerful tool in random matrix models, for example, in the recent proof of the Ring Theorem for a class of invariant non-normal matrix ensembles.
- RMT has emerged as a key tool in multivariate statistics in the case where the number of variables and the number of samples is comparable and large. For example, there are now major applications of RMT to population genetics via principal component analysis.
- Over the last year, RMT behavior has been discovered and verified in a set of laboratory experiments on turbulence in nematic liquid crystals.
- There have been major advances in understanding beta-ensembles of random matrices for general beta (alternatively, log-gases at arbitrary temperatures). In particular, the statistics of the spectra of beta-ensembles have been linked in a fundamental way to the statistics of the eigenvalues of a distinguished class of random Schrödinger operators.
- The *Painlevé Project* has been launched. The Painlevé equations play a key role in RMT, but more generally they form the core of modern special function theory. The goal of the project is to foster the study of the properties of the Painlevé functions, algebraic, analytical, asymptotic and numerical, and to collate the information in handbooks, as was done for the classical special functions in the 19th and 20th centuries. (See the opinion piece in the *Notices of the AMS*, December 2010, for more information.)

In addition to the structural developments outlined above, there have been many direct applications of RMT. To give one striking example: the bus delivery system in Cuernavaca, Mexico, was found to obey RMT statistics. The bus system in Cuernavaca (as well as many other cities in Latin America) has certain built-in distinguishing features which are designed to avoid the bunching of buses, as well as long waits between buses.



Distribution of the fluctuating local radius for growing clusters of liquid crystal turbulence. The distribution function is shown with axes properly rescaled by experimentally measured parameter values, so that no fitting is performed. The dashed and dotted curves indicate the Tracy–Widom distribution for GUE and GOE random matrices, respectively. Adapted from K. Takeuchi and M. Sano, *Phys. Rev. Lett.* 104, 230601 (2010), "Universal fluctuations of growing interfaces: evidence in turbulent liquid crystals."

Three workshops were scheduled for the Semester Program at MSRI on RMT, IPS and IS. The first workshop, which took place from September 13 to 17, focused on internal questions in RMT, such as universality, and also on ideas and methods from integrable systems, such as the Riemann–Hilbert Problem and the associated steepest-descent method. The second workshop, the Connections for Women Workshop, took place on September 20 and 21, and in addition to some of the themes in the first workshop, there were also talks on free probability and random graph theory. The third workshop is due to take place from December 6 through 10, and will focus mostly on the connections between RMT and random growth processes.

Several textbooks on RMT have appeared since 2000: besides one by the author and Dimitri Gioev (2009, Courant Institute), we mention *Random matrix theory*, by P. J. Forrester, N. C. Snaith, and J. J. M. Verbaarschot (2003); *Random matrices* by M. L. Mehta (2004); *An introduction to random matrices* by G. W. Anderson, A. Guionnet, and O. Zeitouni (2010); *Skew-orthogonal polynomials and random matrix theory* by Saugata Ghosh (2009); and books on the application of random matrix theory to a variety of fields such as wireless communications (A. M. Tulino and S. Verdú, 2004) and multivariate statistics (Z. Bai, Y. Chen, and Y.-C. Liang, 2009) — not to mention numerous conference proceedings.

Key problems and conjectures remain unresolved. Prime examples are the behavior of random band matrices and the question of universality for Last Passage Percolation-type models with arbitrary independent waiting times. New universality classes have emerged (Pearcey and beyond!) and new applications of RMT continue to appear with an astonishing regularity. We look forward to the coming decade in RMT with great anticipation.

Focus on the Scientist: Percy Deift

Jinho Baik and Gérard Ben Arous

Percy Deift is now the heir of a long tradition of classical and spectral analysis at New York University's Courant Institute of Mathematical Sciences. In the 1980s Percy became involved in the investigation of eigenvalue algorithms (with Luen-Chau Li, Tara Nanda and Carlos Tomei), and later singular value algorithms (with James Demmel, Luen-Chau Li and Carlos Tomei). The focus of these investigations was the relationship between such algorithms and Hamiltonian dynamical systems. The culmination of these investigations was the proof that the QR and Toda algorithms on full $n \times n$ real matrices are each a completely integrable Hamiltonian system.

Among Deift's contributions is a systematic development (in collaboration with Xin Zhou) of the asymptotic analysis of the Riemann–Hilbert problems in the early 1990s. This powerful method has yielded strong precise asymptotic results of integrable systems. Even more remarkably, Deift and his numerous collaborators pushed the influence of their newly developed tools far outside their original domain of classical analysis. Even though the Riemann–Hilbert approach is rather difficult and deep, Percy and his collaborators have very rapidly made it relevant and even indispensable to a very broad set of mathematical questions. Percy has preached by example and has himself successively applied this Deift–Zhou method, as it is now known, to a wide range of problems with the help of his students and collaborators.

The most widely acclaimed success has probably been the solution of the long standing conjecture by Ulam about the longest increasing subsequence of a random permutation (obtained with Jinho Baik and Kurt Johansson in 1999). This has established a bridge between the Tracy–Widom fluctuations for the largest eigenvalue of random matrices and a large class of combinatorial problems and limit theorems for particle systems that fall in the Kardar–Parisi–Zhang universality class of statistical mechanics. This universality class is wide and contains many important growth processes and interface fluctuations in statistical physics.

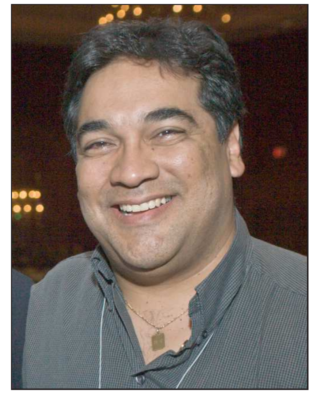
The domain of relevance of the Riemann–Hilbert techniques developed by Percy and his collaborators now intersects many more important domains of mathematics: integrable PDEs (the long-time behavior of the Korteweg–de Vries equation and nonlinear Schrödinger equations), perturbation theory of nonlinear Schrödinger equations, integrable ODEs (Painlevé equations), random matrix theory (universality of the eigenvalue statistics of the unitary and orthogonal invariant ensembles), combinatorics (as mentioned above: the limiting distribution of the length of the longest increasing subsequence of a random permutation) and, naturally, orthogonal polynomials (asymptotic behavior of general orthogonal polynomials).

Percy Deift's numerous honors include the George Pólya Prize of SIAM (1998), Guggenheim Fellowship (1999), an invited address at the ICM (Berlin, 1998), a plenary lecture at the ICM (Madrid, 2006), a plenary lecture at the ICMP (Rio de Janeiro, 2006), and the Gibbs Lecture (2009). Percy has been a member of the US National Academy of Sciences since 2009, and a fellow of the American Academy of Arts and Sciences since 2003.

A Word from the Director

(continued from page 1)

the mathematics community. They learned how MSRI has worked to increase diversity (including participation from underrepresented groups and women) among participants and members, that the scientific programming at MSRI is consistently of the highest degree, that it hosts leading mathematicians from around the world to meet and collaborate at the Institute (in numbers that nearly overwhelm the physical capacity of the building), and of the mentoring program between senior mathematicians and postdocs that enriches their work and careers.



We felt that the Site Visit went very well and that the Site Visit Team had been presented a thorough account of the Institute's activities and the effectiveness of our programs. In late December 2009, the team sent us their report, along with a request for further information regarding a few issues, which we were happy to provide.

In mid-March of this year, the Institute Management Team of the DMS at the NSF let us know that our proposal was being recommended to Congress for renewal of another five-year period at a funding level significantly greater than the Institute has received in the past. Knowing the exact level of anticipated NSF funding for the next five years allowed us to submit a more fine-tuned budget, and this revision was approved by the NSF in late August.

I want to acknowledge and thank my staff, especially Hélène Barcelo, and our Board of Trustees for their many contributions and hard work to make our proposal successful.

The strength of MSRI and its success in serving the mathematics community rests on the generous help that we receive from its leaders. We are enormously grateful for their donation of time and expertise and are always pleased when their contributions are recognized by others.

Thus, it's a pleasure to congratulate George Papanicolaou, who serves on our Scientific Advisory Committee, on his having been awarded this past June the first William Bentner Prize in Applied Mathematics, with its impressive diploma (above). The citation for this prize mentioned his "outstanding contributions in mathematics linking theory to applied problems in various areas including imaging analysis."

Ricardo Cortez (top right), cochair of our Human Resources Advisory Committee, received the 2010 SACNAS Distinguished Undergraduate Institution Mentor Award this August. He was cited for his work with minority undergraduates, including his leadership in helping found MSRI-UP, our summer undergraduate program, which has been going strong and getting stronger since 2007.

It's also a pleasure to congratulate Jean Bourgain, of the Institute for Advanced Study, who is visiting MSRI regularly this year as a member of our Complementary Program, on his being awarded the Shaw Prize in the Mathematical Sciences this September for his groundbreaking work in mathematical analysis.

Finally, if you have not yet done so, please come visit our new web site, launched in October. We are excited about the new capabilities of the web site to foster new modes of communication between MSRI, its members, mathematicians, and the general public.

New Staff Members

MSRI is excited to have two new staff members in key positions: Jennifer Sacramento is now our Program Coordinator, and Phyllis Carter (far right) is our Chief Financial and Administrative Officer.

Marsha Borg

Phyllis is responsible for coordination and management of financial, human resources and administrative operations at MSRI. She brings to MSRI over 25 years of experience as a senior level manager at both for-profit and nonprofit organizations. Most recently, she was CFO of Playworks, Inc., a national nonprofit focused on youth education and health. At Playworks, she led the Finance and IT department through a national expansion from 5 to 11 cities across the US. Phyllis has an MBA from Washington University Olin School of Business in Saint Louis and lives in Oakland. She loves travel, volunteering in youth education and hiking Bay Area peaks.

Jennifer comes to us from The Gubbio Project, a homeless day-shelter in San Francisco, where she served as Executive Director and Board member. Jennifer has been putting her experience to excellent use in providing hospitality and support to all the visitors of MSRI and the Greater Bay Area.

Inverse Problems and Applications

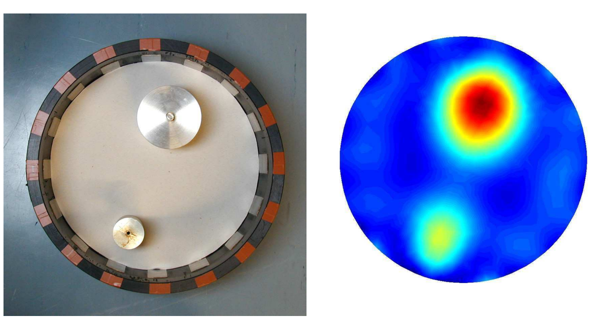
Liliana Borcea, Maarten de Hoop, Peter Kuchment and Gunther Uhlmann

In inverse problems one probes a medium, or an obstacle, with a particular type of field and measures the response. From these measurements one aims to determine the medium properties and/or geometrical structure. Typically, the physical phenomenon is modeled by partial differential equations and the medium properties by variable, and possibly singular, coefficients. The interaction of fields is usually restricted to a bounded domain with boundary: a part of the human body, the solid earth, the atmosphere, an airplane, etc. Experiments can be carried out on the boundary, and the goal is thus to infer information on the coefficients in the domain's interior from the associated boundary measurements. The key questions addressed in inverse problems concern the unique identifiability of the coefficients, the stability, and explicit reconstruction, under assumptions of complete or partial boundary data.

The mathematical techniques needed to study inverse problems are diverse, and include those from the analysis of partial differential equations, microlocal analysis, abstract and applied harmonic analysis, complex analysis, integral geometry, differential geometry, algebraic geometry, control theory, optimization, stochastic analysis, and discrete mathematics.

Calderón's problem

Calderón's inverse problem, which forms the mathematical foundation of electrical impedance tomography (EIT), is a fundamental example of an inverse problem. In it, one wishes to determine (if possible) an unknown conductivity distribution inside a bounded domain modeling — for example the earth, a human thorax, or a manufactured part — based from voltage and (static) current measurements made on the boundary. The initial motivation to propose this problem came from geophysical prospecting. In the 1940s, before his distinguished career as a mathematician, Calderón was an engineer working for the Argentinian state oil company, Yacimientos Petrolíferos Fiscales. Apparently, at that time Calderón had already formulated the problem that now bears his name, but did not publicize his work until 30 years later.



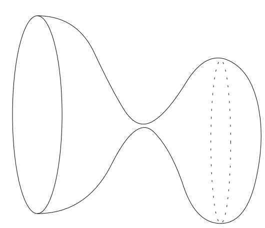
In medical imaging applications, EIT strives to recover internal conductivity of tissues by boundary measurements. For example, in the model experiment shown in the figure above, the conductivity data can be processed to give the approximate reconstruction on the right (courtesy J. Kaipio, Finnish Center of Excellence in Inverse Problems). One widely studied potential application of EIT is the early diagnosis of breast cancer.

EIT is an example of a very diffuse inverse problem. The currents go instantaneously everywhere in the medium (as opposed, for example, to X rays where electromagnetic energy propagates along straight lines). Conductivities can be anisotropic, that is, depend not only on position but also on direction. An example is muscle tissue in the human body.

Calderón's problem has been extensively studied in the last 30 years, and there have been intricate solutions to many of its appearances, concerning the regularity of the conductivity and the extent of the data. Still, there remain fundamental questions unresolved, such as the uniqueness modulo change of variables for the anisotropic conductivity case, and partial boundary data, in dimension three.

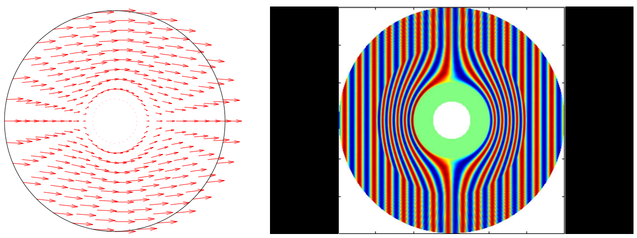
Cloaking

Electrostatics. In studying Calderón's problem, conductivities were found that cannot be distinguished from a constant conductivity by making electrostatics measurements at the boundary of a given domain. The idea is to use the fact that the equation describing the potential in the case of electrostatics is invariant under changes of coordinates. In dimension three, conductivities can be identified as Riemannian metrics, and one can formulate Calderón's problem as determining the Riemannian metric of a manifold with boundary by measuring the Dirichlet-to-Neumann map at the boundary of the manifold. Indeed, the problem, being of geometric nature, is invariant under changes of coordinates or transformations that leave the boundary fixed, which opens the way to develop an approach to make objects invisible.

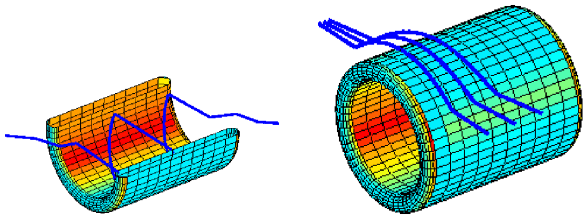


The preceding drawing shows a surface (representing a manifold in the higher-dimensional case) where the "neck" is pinched. In the limit, the manifold has a pocket about which the boundary measurements do not give any information. If the collapsing of the manifold is done in an appropriate way, in the limit, we obtain a (singular) Riemannian manifold which is indistinguishable from a flat surface. This can be considered as a conductivity, singular at the pinched points, that appears to all boundary measurements the same as a constant conductivity.

Invisibility Invisibility has been a subject of human fascination for millennia, from the Greek legend of Perseus versus Medusa to the more recent *The Invisible Man* and *Harry Potter*. Since 2005 there has been a wave of serious theoretical proposals in the physics literature for cloaking devices – structures that would not only render an object invisible (light rays) but also undetectable to finite-frequency electromagnetic waves. A proposal that has received particular attention, because in principle it can cloak any object of any shape and size, is that of Pendry and coworkers.[1] It has been referred to in the physics literature as *transformation optics*. Essentially the same idea was formulated earlier in electrostatics terms: a singular transformation[2] is used to blow up a point to a sphere, forming the boundary of the cloaked region. Pushing forward a constant conductivity using this transformation gives an anisotropic conductivity, whose currents have the behavior illustrated in the next figure. No current flows in the (inner) ball of radius 1, making this region effectively invisible to boundary measurements. All the electrostatics measurements made on the boundary of the ball of radius 2 are the same as the case of homogeneous conductivity.



Other constructions using transformations have been proposed; we mention *electromagnetic wormholes*. The idea is to trick electromagnetic waves to think they are going through a handle. Using an electromagnetic wormhole one can create a secret connection between two points in space. One knows the “input” and can encode the “output”. The wormhole itself is invisible.



The blueprint of electromagnetic parameters used for cloaking have not been found in nature. Indeed, there is a very active area of research in *metamaterials* to construct cloaking devices. In a widely reported experiment[3] this has been accomplished at microwave frequencies. As stated in *Science*, the theoretical ideas for cloaking based on mathematical transformations have produced and will produce a long shadow.

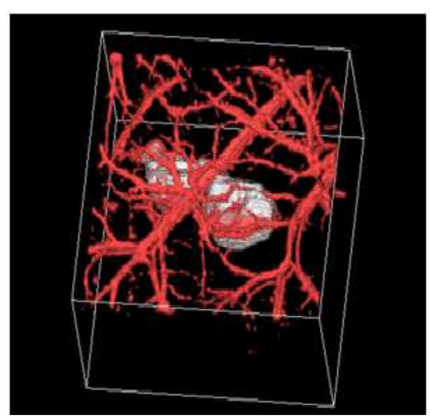
Medical imaging

Many techniques of medical imaging exist today, and many have motivated extensive mathematical analyses. In medical imaging, one naturally exploits different modalities to resolve a range of physiological parameters. However, they also differ in their sensitivity, resolution and safety.

The oldest medical imaging application is the now standard *X-ray computed tomography* (usually called CT-scan). The goal here is to reconstruct tissue density in the interior of a domain from observations of the degree of attenuation of thin beams of X rays, passing through the domain. Mathematically, here one strives to recover a function of two or three variables from its integrals over “all” straight lines, which is called the Radon transform. It was in the 1950s and 1960s that the first X-ray CT scanners for medical imaging were developed, for which eventually A. Cormack and G. Hounsfield received in 1979 the Nobel prize in medicine. Even though the CT scan is by now a well established technique, the more recent technological developments yielding truly three-dimensional probing (that is, beyond the slice-by-slice procedure) poses new mathematical challenges concerning the reconstruction of density.

In *emission tomography*, one aims to detect the internal distribution of radiation sources in a nontransparent body. The patient is injected with a small dose of radioactive substance and then its distribution is monitored by a scanner. The inverse problem here also leads to the inversion of a Radon type transform, but now with a rather complicated functional weight in the integrand. Though the basic questions — uniqueness of reconstruction from such data and inversion — have been resolved recently, some important mathematical issues concerning this modality remain open.

We mention a few very recent techniques in medical imaging, some of which are still subjects of thorough mathematical and engineering investigations. In *ultrasound vibro-acoustography* one focuses two beams of ultrasound at two slightly different high frequencies on the region of interest. The two fields interact nonlinearly there, to create a force at the (low) difference frequency, detectable on the boundary. The inverse problem is related to the force. In hybrid methods, one aims to combine phenomena of different physical natures (for example, electromagnetism and ultrasound) to overcome their individual deficiencies and combine their advantages, so long as they are coupled. Examples include *thermo/photo-acoustic tomography*, where one heats up the tissue with a brief electromagnetic pulse and uses ultrasound transducers to “listen” to the resulting acoustic wave. (The figure shows the photo-acoustic image of



Wikipedia / L. Wang

[1] J. B. Pendry, D. Schurig and D. R. Smith, “Controlling electromagnetic fields”, *Science* **312** (June 23, 2006), 1780–1782. A related idea is given in U. Leonhardt, “Optical conformal mapping”, *Science* **312** (23 June, 2006), 1777–1780.

[2] A. Greenleaf, M. Lassas and G. Uhlmann, “On nonuniqueness for Calderón’s inverse problem”, *Math. Res. Lett.* **10**:5–6 (2003), 685–693.

[3] D. Schurig, J. Mock, B. Justice, S. Cummer, J. Pendry, A. Starr and D. Smith, “Metamaterial electromagnetic cloak at microwave frequencies”, *Science* **314** (2006), 977–980.

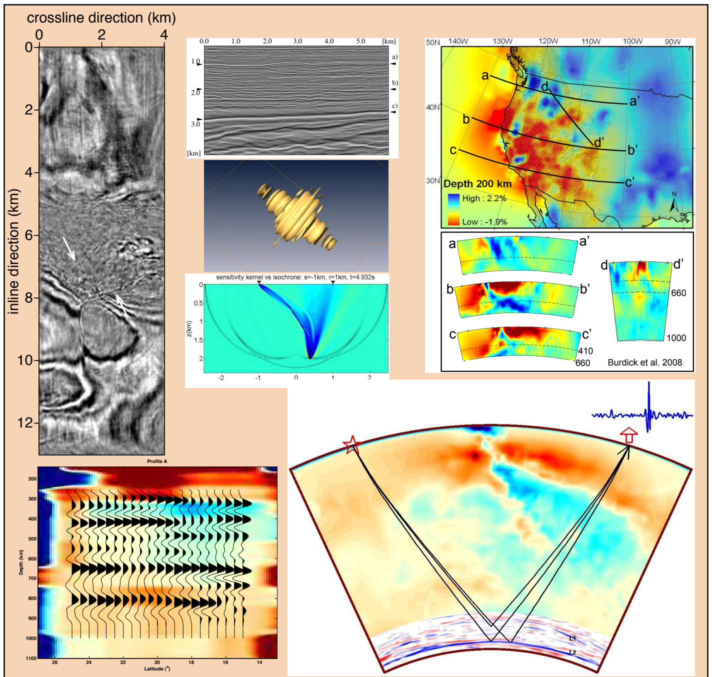
a melanoma.) The idea is to exploit the high contrast from electromagnetic absorption and high resolution from the ultrasound. Many mathematical issues here are unresolved. Topics of current interest, each of which involves demanding mathematics, include fully 3D X-ray CT, elastography, fluorescent tomography, electron microscope tomography, magnetic resonance elastography, and various hybrid methods.

Geophysical inverse problems

Inverse problems in geophysics aim to image and constrain source locations and mechanisms, and material properties and the geology and sedimentary processes in the earth's crust, as well as the geochemistry, mineral physics, and geodynamics on a planetary scale. Though there are various important new results pertaining to potential fields, such as in gravity gradiometry and geomagnetics, we focus here on the category of waves (seismic, ground-penetrating radar) and diffuse (electromagnetic) waves. Also, in geophysics, hybrid modalities have been considered and studied, in particular, the electroseismic effect. Here, we discuss various aspects pertaining to solid earth geophysics, including applications in hydrocarbon exploration and production. We just mention the many developments which have taken place, for example, in hydrologic inversion concerning contamination problems such as determining a pollution source, and remote sensing from space.

A full description of seismic waves is given by a system of elastic-gravitational equations including the effects of the rotation of the earth. On short time scales these are typically approximated by a system of (anisotropic) linear partial differential equations describing elastic waves. Through arguments of polarization decoupling, in many applications, these equations are further approximated by scalar or acoustic wave equations. The coefficients of these partial differential equations naturally vary on a wide range of scales, and in particular regions may contain fluctuations described by various random processes; they capture the material properties, microstructure, and the above mentioned processes. Seismic waves can be excited by various natural (earthquakes, microseismicity, tremor, ambient noise) and controlled (marine, land) sources. The wave field is observed in large (dense) arrays of sensors (such as USArray) at the earth's surface (or streamer and ocean bottom cables in the marine setting). The locations of sources and receivers form the acquisition geometry and control the illumination of the subsurface. From these measurements one can in principle extract or recover spectral data (normal modes), transient phases, and distinguish dispersive surface waves (upon describing the subsurface locally by a bent slab in a half space). In certain cases, one can construct the Neumann-to-Dirichlet map on an open set from the measurements in the given acquisition geometry. In any case, the mappings from the mentioned coefficients to the data is nonlinear and the study of their properties is one of the main subjects in inverse problems. The linearized problems lead to the introduction of imaging operators and are often studied via normal operators.

In many seismic applications, observations typically have to be mapped to data such as travel times, finite-frequency travel times, wave form polarized phases, the Neumann-to-Dirichlet map, etc. Examples of images and reconstructions are given in the sidebar.



Left – top: a horizontal section of a reconstruction using singly scattered compressional waves showing salt tectonics in the North Sea; bottom: a horizontal section of a reconstruction of phase transitions in Earth's transition zone near Hawaii using *SS* precursors (earthquake data); the depths of these are used to infer the presence and location of plumes. Middle – top: a horizontal section of a reconstruction (showing sedimentary layers) using singly scattered elastic waves (*qPqP* and *qPqSV* reflections) in anisotropic models in the North Sea; middle: a wave packet, used, for example, in data compression and regularization and partial reconstruction; bottom: combined sensitivities using a single (finite-frequency) data sample in reflection tomography and inverse scattering, in the presence of caustics. Right – top, middle: horizontal and vertical sections of a reconstruction based on the geodesic X-ray transform using *P* waves and USArray data; bottom: a section of an image showing the subducting Cocos plate, the D″ layer, post-perovskite phase transition and the core-mantle boundary.[3]

Surface-waves tomography. In surface-wave tomography, one localizes the upper structure of the earth and views it as a (curved) slab or half space. The typical strategy follows a high-frequency, semiclassical analysis in which the medium variations in the direction normal to the surface of the slab are rapid as compared to the variations in the transverse directions. In this case, the problem can be described in terms of propagation along the slab's surface tied to locally one-dimensional spectral problems reflecting the material properties underneath any point on the surface. In the inverse problem, propagation over the surface yields pointwise frequency-dependent phase velocities which are then studied, and interconnected, in the mentioned spectral problems.

Travel tomography: body-wave phases. The classical inverse problem in seismology is travel tomography. The first inverse formulation and reconstruction is due to Herglotz and Wiechert.

Here, travel times are viewed as the boundary distance function of a Riemannian manifold (the earth) with boundary. The material properties are assumed to vary smoothly. Thus the polarizations of elastic waves can be decoupled. Uniqueness results were initially established for the isotropic case and simple metrics, that is, in the absence of caustics. In the anisotropic case, one obtains uniqueness of reconstruction up to a change of variables that is the identity near the boundary. In dimension two the anisotropic case for simple metrics has been fully analyzed through a connection to Calderón's problem. In dimension three, also, the case of simple metrics has been understood. Local uniqueness has been established in the presence of caustics using the scattering relation as the data.

In global seismic applications, one typically considers linearization, that is, the geodesic X-ray transform. The known injectivity results for this transform are very similar to those for the nonlinearized case. The lack of injectivity in the presence of caustics (focusing and defocusing) underlies the unconstrainedness of thermochemical convection in the Earth's mantle (think plumes). The normal operator associated with the restricted X-ray transform belongs to an $I^{p,l}$ class metric with constant curvature.

Progress in admitting the formation of caustics has been based on incorporating additional information from the observations. In the presence of caustics, more recent results have been obtained using the scattering relation as the data. The scattering relation contains the slope information (direction of the ray hitting the surface) which is revealed in seismology by so-called vespagrams and beamforming.

Avoiding the difficulty (and perhaps impossibility) of picking travel times in finite-frequency observations, in seismology, one has replaced the differential time data in the geodesic X-ray transform by cross correlations of the trace of the solution in the reference model with the observations. The data become the locations of the maxima of the cross correlations nearest to the origin. The approach has been coined wave-equation tomography. Thus the formulation is based on partial differential equations. We are just beginning to understand the map that represents the analogue of the geodesic X-ray transform. Wave-equation tomography opens the possibility of considering coefficients of reduced regularity. However, the question of injectivity remains open.

One of the focus areas in seismic tomography has been the unravelling of anisotropy, to analyze deformation and subduction processes, for example through lattice-preferred orientation, constrained by GPS measurements. The strategy is based on shear-wave splitting, and one has used differential travel times and splitting intensities as the data. Many questions remain open about ways to uniquely determine the local orientations of symmetry axes of the stiffness tensor in the subsurface.

Separable inverse problem: single-scattered phases. Imaging discontinuities using singly scattered phases in the data has a long history and dates back to the work of Hagedoorn. The point of departure is a separable inverse problem in which the modeling of the data is linearized about a reference. (There are no proper estimates for this linearization.) The idea is that the reference is smooth and varies on a coarse scale, whereas the perturbation, or contrast, captures the singularities and varies on the finest scales.

The inverse problem of determining the reference / background is essentially "reflection" tomography or "velocity" inversion which is strongly nonlinear; however, it is commonly dealt with using local optimization strategies (no proofs of convergence). The inverse problem of determining the contrast is inverse scattering and, naturally, ignores multiple scattering. Roughly speaking, there are two directions to realize reconstructions: The reverse-time approach, and the reverse-depth (or downward continuation) approach (which can be formulated in curvilinear coordinates leading to the notion of pseudodepth).

The linearization is identified as the Born or the Kirchhoff approximation and defines a single scattering operator mapping the contrast to the data on the boundary. The Kirchhoff approximation distinguishes itself by honoring the boundary conditions across surfaces of discontinuity. The calculus of Fourier integral operators has provided a deep insight in the inverse scattering problems. Indeed, under weak conditions the single scattering operator is a Fourier integral operator. The challenges of imaging appear through the so-called acquisition geometry limiting where sources and receivers can be placed. A maximal geometry has dimension $2n - 1$. A codimension-2 situation occurs in the common source acquisition typical for earthquake data, while the codimension-1 situation is typical for marine streamer acquisition. The normal operator in all these cases is pseudodifferential, that is, does not generate artifacts, if the canonical relation of the single scattering operator satisfies the Bolker condition. However, in the presence of caustics, this condition can be violated. In particular cases, the normal operator belongs to an $I^{p,l}$ class and the generation of artifacts can be characterized.

The key test whether the reference is estimated sufficiently well is the "range test" for the single scattering (or modeling) operator (the adjoint of the inverse scattering operator): To check whether the (presumably singly scattered) data are in the range of the single scattering operator, one can derive an annihilator (which depends on the reference model). With sufficiently many scatterers, one can prove uniqueness of the reflection tomography problem. However, it is all based on single scattering theory and separation of scales.

It is common practice to carry out elastic-wave polarization decoupling. Naturally information is contained in the polarization coupling. Mode conversions have been exploited in the imaging of discontinuities without knowing the source. The idea is to use P to S conversions and image their cross correlations in the data. Seismologists refer to this approach as receiver functions.

Annihilators are also closely related to elliptic minimal projectors (onto the range of the mentioned single scattering operators) which can be exploited, under certain conditions, to fill in missing data or generate a desirable acquisition geometry. Partial reconstruction (accounting for partial illumination) is naturally formulated by representing the mentioned single scattering and normal operators as matrices using a tight frame of curvelets. Here one exploits sparsity of these matrices, and the observation that the data and geological images can be compressed using curvelets.

Full-wave methods. With the state of supercomputing, reconstruction techniques using the full waveform data, in dimension 3, have become feasible and hence have received significant attention. Here, for example, computational linear algebra has played an important role. Most developments have been focused on obtaining local results and approaches which assume reconstruction close to the actual model, using various techniques from optimization and regularization. One class of results pertain to the multifrequency formulation using the Helmholtz equation. Local convergence of iterative methods is being studied, which is closely connected to the stability analysis of the inverse problem. The availability of low-frequency data and dual-sensor data (detecting the boundary normal derivatives of the wave field) appear to be important in this context. Moreover, various approaches are being explored to generate representations of the Neumann-to-Dirichlet map from the observations on parts of the boundary.

The inverse problem of the single-frequency Helmholtz (or Schrödinger) equation using the Dirichlet-to-Neumann map as the data has been extensively studied, with different conditions on the regularity of the reciprocal wave speed or potential. With full boundary data, uniqueness was established for bounded and measurable potentials. (There is a natural connection to Calderon's problem through a mapping of conductivity to a potential.) The proofs of this and many other results make use of the construction of Complex Geometrical Optics solutions. The key problem of partial boundary data with, essentially, sources and receivers on a common open set remains open.

In the case of the scalar-wave equation, uniqueness has been established using methods of time-domain boundary control and unique continuation. However, the corresponding problem for systems, for example, describing elastic waves, remains open.

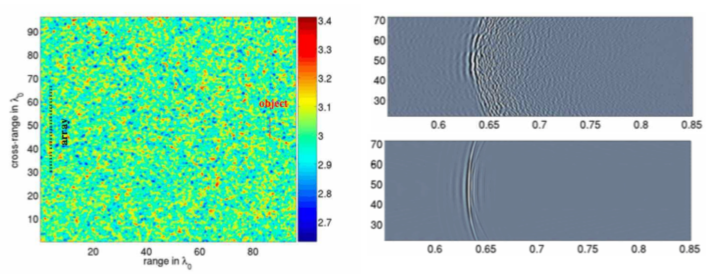
Controlled source electromagnetic method. As an example of diffuse waves, we mention the inverse problem derived from the low-frequency Maxwell equations. In the magnetotelluric method, naturally occurring electric and magnetic fields are used, while in the controlled-source electromagnetic (CSEM) method one generates fields with point electric and magnetic dipoles on the boundary of the manifold representing the subsurface. Marine electromagnetic methods are, for example, used to characterize magmatic and hydrothermal systems in fast spreading mid-ocean ridges. In exploration, CSEM is used for direct hydrocarbon detection. One can cast this problem in the form of a Schrödinger equation with matrix potential. The data are represented through an impedance map. Uniqueness was established for C^4 conductivities with full-boundary data, again, using complex geometrical optics solutions.

Random media and noise sources

Noise interferometry. Relatively recent work on time reversal of waves in a random medium has shown that medium fluctuations are not necessarily detrimental to, but may in fact enhance various operations with waves. In interferometry, one considers "field-field" cross correlations associated with (ambient) noise observed at pairwise distinct receivers, to obtain an "empirical" Green's function, which process is naturally related to time reversal. Indeed, results have been obtained rigorously, where the cross correlation yields the Green's function up to an integral operator the kernel of which is described by an Ito–Liouville equation, which admits, under certain conditions, statistically stable solutions. Indeed, better estimates (when the Green's function is better resolved) may be obtained in a randomly inhomogeneous medium than in a deterministic homogeneous medium, as a consequence of the wider angular spread in the phase-space representation of a wave in the random medium. The enhanced resolution occurs due to an exponential damping factor that appears in the analysis of the cross correlation, and that involves the structure function of the medium. The cross-correlation technique has been successfully applied perhaps most notably to the Apollo 17 Lunar Seismic Profiling Experiment. The correlations were used in an inverse problem estimating the thermal diffusivity in the shallow lunar crust, while heating from the Sun is the ultimate cause of the seismic noise.

Effectively using receivers as sources through the mentioned "field-field" cross correlations, one can generate, in principle, a rich set of data or even a Neumann-to-Dirichlet map on part of the surface (boundary of a manifold describing the subsurface), even where deterministic sources are necessarily absent. While current studies relating to the heterogeneous earth mostly make use of surface-wave contributions to the Green's function estimate, the importance of understanding the behavior of (scattered) body waves has been recognized.

Imaging and clutter. The goal of sensor array imaging is to create maps of the structure of inaccessible media using sensors that emit probing pulses and record the scattered waves, the echoes. We call the recorded echoes array data time traces, to emphasize that they are functions of time. Because the array has finite size and the data is band limited, we cannot determine in detail the medium structure, and the inverse problem must be formulated carefully to be solvable. In general, we distinguish between determining singularities in the wave speed, which arise at boundaries of reflectors, and the background speed. The latter determines the kinematics of the data, the travel times of the waves, and the former is responsible for the dynamics of the data, the reflections. Array imaging is typically concerned with locating the reflectors in the medium, but in order to be carried out it requires knowledge of the background wave speed, or its determination by other methods.



Left: Setup for imaging a crack in a random medium. The color bar indicates the fluctuations of the wave speed in km/s. Right: Time traces of the echoes received at the array in the random medium (top) and in the homogeneous medium (bottom). The abscissa is time in ms and the ordinate is the receiver location in the array, in units of the central wavelength.

Focus on the Scientist: Liliana Borcea

George Papanicolaou

ipam.ucla.edu

Liliana Borcea started her research in inverse problems with her Stanford thesis on high-contrast impedance tomography. Around 2000, she moved to array imaging, especially imaging in random media, but she has been very active in impedance tomography as well, with her pioneering work on optimal grids done jointly with her PhD students Fernando Guevara Vasquez (now at Utah) and Alexander Mamonov, and with Vladimir Druskin of Schlumberger Research.

In array imaging she has introduced and developed extensively coherent interferometric imaging, CINT, (along with George Papanicolaou and Chrysoula Tsogka), which is an image formation algorithm that is based on cross correlations of array data, rather than the data itself as is usually done. CINT is robust in complex and unstable environments but it must properly assess and adapt to such environments. It is this aspect of CINT that is mathematically most challenging. She has introduce many other important innovations in array imaging, such as optimal illumination techniques using generalized prolate spheroidal functions, the use of the local cosine transform for filtering data coming from strongly inhomogeneous media, special data filtering algorithms for imaging in layered media, etc. In the last two years she has also begun an in-depth analysis of autofocus and motion estimation problems in synthetic aperture radar using phase-space methods.

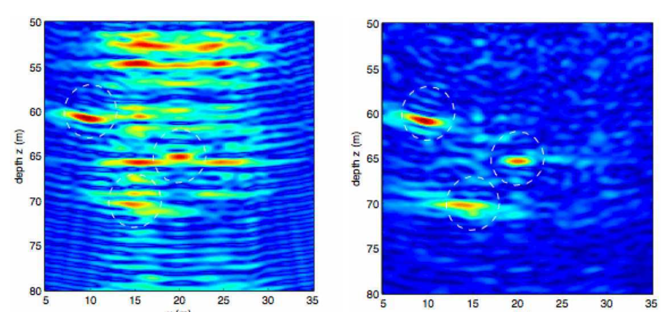
Liliana received her PhD from Stanford in 1996, and then was awarded an NSF postdoctoral fellowship which she held at the California Institute of Technology. Subsequently she moved to Rice University where she is currently a Noah Harding professor. Her (NSF supported) research on imaging in random media was selected by the NSF as a research highlight used in the NSF budget request to Congress.

In the setting of the single scattering theory above mentioned, in many applications, the background medium is cluttered due to the presence of small inhomogeneities that interact with the waves probing the scatterers. Clutter poses serious impediments to the imaging process. In recent years, significant progress has been made in addressing this problem, both from experiments and in developing a comprehensive mathematical framework, which has led to new resolution theories incorporating concepts such as statistical stability. A natural approach to studying waves in cluttered media is to model the unknown inhomogeneities, the clutter, with random processes. Wave propagation in random media has been extensively studied in the framework of stochastic partial differential equations. The success of imaging with clutter relies on a detailed understanding of the cumulative, multiple scattering effects of the inhomogeneities in the medium in order to mitigate them and produce reliable results that are independent of the realization of the medium.

Imaging in moderately backscattering media can be carried out with the coherent interferometric (CINT) method. CINT images are formed using cross-correlations of the array data traces computed locally in time and over sensor offsets. The thresholding in time and space in the computation of the cross-correlations is motivated by the statistical decorrelation of the wave fields at the array, due to multiple scattering in the medium. The CINT imaging function is in fact the smoothed Wigner transform of the data traces evaluated at travel times and directions from the array sensors to the point at which we form the image. The smoothing is controlled by the time and sensor thresholding and it is because of it that statistical stability can be achieved. Stability comes, however, at the cost of loss of resolution; the resolution can be improved using delicately designed data filters to image selectively various parts of the reflectors.

The problem becomes more challenging in the case of strong clutter, where the coherent primary echoes off the reflectors are weak and overwhelmed by the medium backscatter. Recent work shows that, under certain conditions it is possible to detect these coherent echoes and then filter the unwanted clutter backscatter; this is illustrated in the figure: The CINT method alone does not deal with the clutter backscatter and produces the image on the left. However, CINT with filtered data traces produces improved resolution in the figure on the right.

When the multiple scattering by the inhomogeneities is so strong that no coherence is left in the array data, coherent imaging methods cannot be used anymore. That is to say, we cannot form an image by simply adding the time delayed array data traces or their cross-correlations. Instead, the strategy is to resort to parameter estimation problems that are usually based on a transport theory model to describe how the energy propagates in the strongly scattering medium.



Left: CINT image in heavy clutter. Right: CINT image with the filtered data. The three reflectors are indicated with white circles and correctly located.

Puzzles Column

Elwyn Berlekamp and Joe P. Buhler

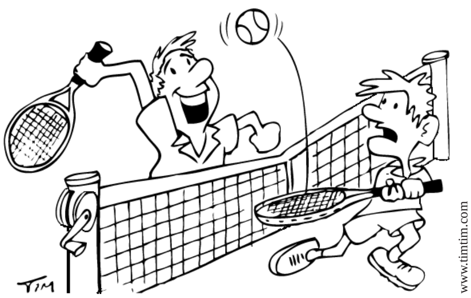
1. (a) Prove or disprove: There is a subset S of nonnegative integers such that every nonnegative integer can be written uniquely in the form $x+2y$ for $x, y \in S$.

(b) Same question with both occurrences of "nonnegative" omitted.

Comment: Part (b) is due to Richard Stong, and it appeared in the 1996 American Mathematics Olympiad.

2. Vertices A, B, and C of an equilateral triangle of side 1 in the plane are given. Denote by AB the line segment from A to B, by $\hat{A}$ the circular arc with center A and endpoints B and C, and by $\hat{B}$ the circular arc with center B and endpoints A and C. What is the radius of the circle that is tangent to the line segment and the two circular arcs?

Comment: The Guardian, a British newspaper, has amusing puzzles each week; this one appeared in the October 16 edition.



3. Alice, Bob, Charlie, and Diane play tennis in sets: two of them play a set and the winner stays on the court for the next set, with the loser replaced by the player who was idle the longest. At the end of the day Alice played 61 sets, Bob played 22 sets, Charlie played 21 sets, and Diane played 20 sets. Who played in the 33rd set?

Comment: This is a variant, due to Stan Wagon, of a puzzle in Dick Hess's *All-Star Mathlete Puzzles*.

4. (a) Find two infinite power series $f(x)$ and $g(x)$, whose coefficients are only 0 or 1, whose product is $1/(1-x)$.

(b) Describe all such pairs.

Comment: Veit Elser poses this problem, and notes that Leo Moser and N. G. de Bruijn asked similar questions earlier.

5. A team of n people play a game described below. They are allowed to have a strategy session the night before, during which the game is completely described, and they can all plan their joint strategy. You might imagine that they all wear name-tags, and might choose to agree on a fixed ordering, e.g., alphabetical. After the strategy session, no further communication of any kind is allowed between the players.

The game takes place the next day. An umpire places (distinct) real numbers on each of their foreheads. Each player sees all of the real numbers except his or her own. Each player has an ambidextrous white mitten and an ambidextrous black mitten, and must place one mitten on each hand (doing so out of sight of the other players). At that point the players are lined up with the forehead numbers in numerical order. Adjacent players join hands. The team wins if and only if each pair of touching mittens is of the same color.

Devise a strategy that guarantees that the team will win!

Comment: Rumor has it that this puzzle circulated at Google recently.

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