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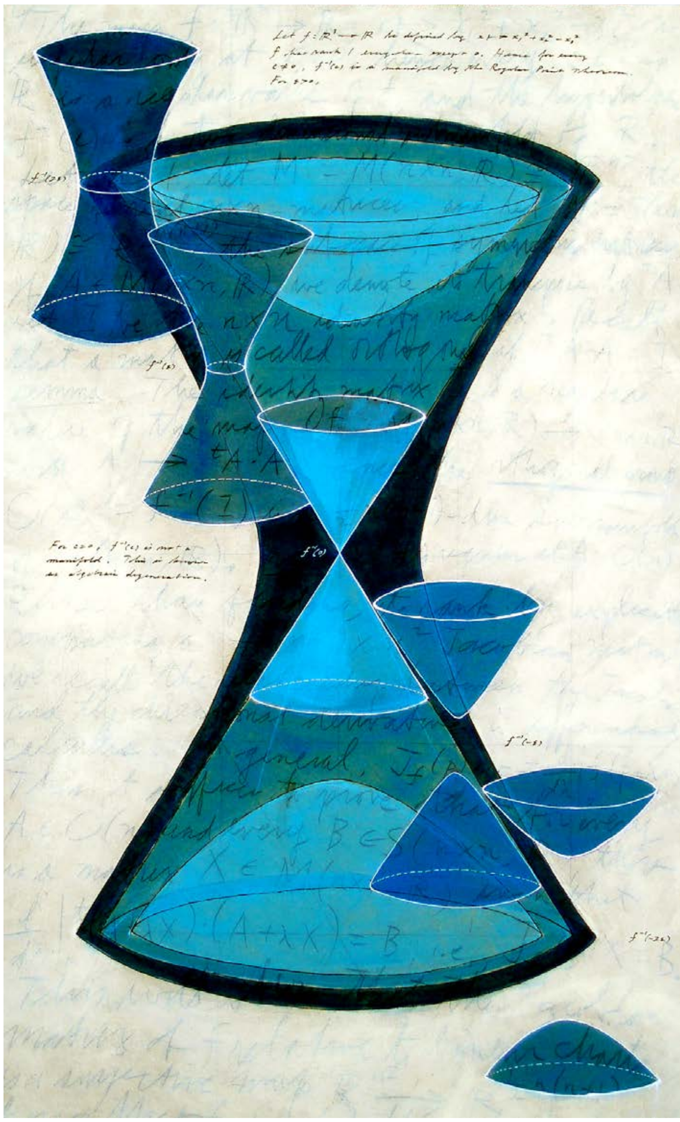


Lun-Yi London Tsai, "Algebraic Degeneration," 2005, acrylic on panel, 39 x 24 in

Geometric Representation Theory. With its artistic representation of geometric representation, Lun-Yi London Tsai's "Algebraic Degeneration" evokes the role of singularities in representation theory, for example in the geometry of adjoint orbits of $SL_2(\mathbb{R})$. See page 8 for the GRT program article.

New Geometric Methods in Number Theory and Automorphic Forms

Pierre Colmez, Steve Kudla, Akshay Venkatesh, and Jared Weinstein

This program aims to highlight new research directions and developments arising through the application of geometric techniques to problems in number theory and automorphic forms.

The branches of number theory most directly related to automorphic forms have witnessed enormous progress over the past five years. Ngô's proof of the fundamental lemma has had far-reaching consequences, for example, for Langlands' program for endoscopy and for the determination of the Galois representations in the cohomology of Shimura varieties. High points in the purely automorphic theory include Arthur's proof of Langlands functoriality for classical groups, using trace formulas that have been established thanks to Waldspurger, Laumon, and Chaudouard, as well as Ngô and Arthur himself.

On the other hand, techniques introduced since 2009 have made it possible to prove new automorphy lifting theorems beyond those developed by Wiles, Taylor and Kisin, and extended to higher dimensions by Clozel, Harris, and Taylor. Combined with the potential modularity method developed by Taylor and his collaborators, these new techniques have had striking consequences, including, for example, the complete Sato–Tate conjecture for elliptic modular forms. Other striking developments in the arithmetic theory of elliptic modular forms include the proof of Serre's conjecture by Khare and Wintenberger and the proof of many cases of the Fontaine–Mazur conjecture (for 2-dimensional Galois representations) by Kisin and Emerton. These developments all make extensive use of the first successes, due especially to Breuil, Colmez, and Berger, in the ambitious attempt to create a p-adic Langlands program in parallel with Langlands' automorphic functoriality

(continued on page 4)

Contents

A River of Activity

David Eisenbud, Director

When I wrote my column a year ago I was again new at MSRI and wrote mostly of things past and things to come. Now I feel very much in the midst of the river of mathematics at the Institute, and of other projects too. Here's a sampling:

Core Mathematical Research

Providing a setting for first-rate mathematical research is the most important thing we do. The two programs at MSRI right now are both closely connected to automorphic forms and the Langlands program. Peter Scholze, from Bonn, is spending the semester at MSRI, and is our Chancellor's Professor. This means that he is giving an advanced course on the campus (http://math.berkeley.edu/courses/fall-2014-math-274-001-lec) where he will explain how to transport Drinfeld's notion of "shtukas," which plays a critical role in the global Langlands correspondence in characteristic p, to fields of mixed characteristic. The program organizers told us that the course could be of historic significance in the area, so we are collaborating with the Berkeley math department to videotape it. You can see the lectures so far (or even follow the course) at https://www.msri.org/people/25488.

This is not the first time that a program in number theory at MSRI has had historic consequences. Thomas Hales (famous for his proof of the Kepler conjecture as well as his work on the Langlands program) was a young member of MSRI's 1986–87 program on Number Theory with Connections to Algebraic Geometry, and he was the first lecturer in the introductory workshop for the Geometric Representation Theory program this fall. In his lecture, he recounted some of the lectures and discussions from 1986–87 about Fermat's theorem: how Ken Ribet's work and discussions with Serre and others concluded with Ken's paper fatefully connecting Fermat to a question about elliptic curves. Wiles was there, too, and subsequently vanished to resurface with a proof of Fermat a few years later. Hales tells the story better than I have — see for yourself at https://www.msri.org/workshops/707/schedules/18786.

Applications of Mathematics

Though MSRI's big programs usually focus on topics from core mathematics, we are also concerned with the applications of mathematics in other disciplines. Our next workshop in this area is Breaking the Neural Code, which will bring a collection of neuroscientists with hard problems about communications within the brain together with mathematicians, statisticians, and computer scientists, who may have interesting techniques to share. The workshop was made possible by the generous support of Trustee Sandy Grossman.

The workshop will start dramatically — in the scientific sense, we hope, but certainly in a musical sense: Sandy Grossman also donated funds to purchase a lovely Steinway piano, and the first day will finish with an inaugural concert by the young virtuoso Theodora Martin. Her program of Chopin, Beethoven, and Brahms will give the instrument a good workout!

MSRI's next applied workshop will be the annual conference on mathematical finance and economics that we run jointly with the Chicago Mercantile Exchange, on the occasion when the CME Group–MSRI Prize is awarded. The workshop topic is "Bubbles in the Market: Why they form and when they burst," a subject considered by the prize-winner, José Scheinkman, in his recent book, *Speculation, Trading, and Bubbles*. The book, by the way, is a little gem, a pleasure to read.

An application of mathematics that borders on play is the theory of Coupon Go, a mathematical variant of the famous game, invented by Elwyn Berlekamp. On February 15, MSRI will be host to two of the great Chinese Go players, Jujo Jang (now retired from professional play) and his wife Naiwei Rui the international women's champion, who will bring a group of aspiring young players for a Coupon Go tournament. Berlekamp and others will speak about the game, and we're told that Chinese TV will be in attendance. If you're a Go enthusiast you won't want to miss this!

CME Group–MSRI Prize in Innovative Quantitative Applications

José A. Scheinkman

The ninth annual CME Group–MSRI Prize in Innovative Quantitative Applications will be awarded to José A. Scheinkman, the Edwin W. Rickert Professor of Economics at Columbia University, the Theodore A. Wells '29 Professor of Economics (emeritus) at Princeton University, and a Research Associate at the National Bureau of Economic Research.

The prize will be awarded at a luncheon in Chicago. Prior to the lunch and award presentation, a panel discussion on "Bubbles in the market: Why do they form, when do they pop?" will be held.

The annual CME Group–MSRI Prize is awarded to an individual or a group to recognize originality and innovation in the use of mathematical, statistical or computational methods for the study of the behavior of markets, and more broadly of economics. You can read more about the CME Group–MSRI Prize at www.msri.org/general_events/21197.

Cha–Chern Scholar

Yaping Yang

Yaping Yang, a postdoctoral scholar in the Geometric Representation Theory program, is the Fall 2014 Cha–Chern scholar.

Yaping was an undergraduate at Zhejiang University in Hangzhou, China. She then joined the graduate program at Northeastern University in Boston, from which she obtained her Ph.D. in Spring 2014 under the supervision of Valerio Toledano Laredo. After her stay at MSRI, Yaping will be moving to UMass Amherst to take up a three year Visiting Assistant Professorship.

Her broad research interests encompass representation theory, algebraic geometry and topology. She has in particular worked on the elliptic cohomology obtained from Levine and Morel's algebraic cobordism. She has also constructed very interesting new monodromy representations of the elliptic braid group of a complex semisimple Lie algebra $\mathfrak{g}$. These arise from finite-dimensional representations of the corresponding rational Cherednik algebra, and from those of a deformation of the universal central extension of the double current algebra $\mathfrak{g}[u,v]$ recently introduced by N. Guay.

The Cha–Chern scholarship combines two funds that were established by Johnson Cha and the family of Shiing-Shen Chern. Both funds are designated to support Chinese Scholars during their stay at MSRI. Shiing-Shen Chern was an outstanding contributor to research in differential geometry and was one of the three founders of MSRI and acted as its first director from 1981–84. Johnson Cha served on MSRI's Board of Trustees from 2000–04. Mr. Cha is the Managing Director of The Mingly Corporation, a Hong Kong investment company.

Application to renew MSRI's primary NSF grant

This one grant currently pays about two thirds of what it takes to run MSRI. The renewal process lasts about a year: we started writing the proposal last January; submitted it in March; got copies of the (very positive) mail reviews in May; and had our site visit in September. The last stages are still to unfurl: we're waiting, as I write at the end of September, for the report of the site visit committee, after which we'll presumably have a budget negotiation sometime around the end of 2014. As I'm sure the reader can imagine, this has taken a great deal of effort and time; but the comprehensive review of MSRI's doings that is part of the process does help inform and sharpen what we do, so there's a significant benefit beyond (we hope!) getting the grant.

Outreach and Education

A year ago, the Simons Foundation gave MSRI a three-year grant to explore outreach events of national scope, and I want to report on four of these:

Numberphile. Since January, MSRI has supported Brady Haran's "Numberphile" channel on YouTube. The channel recently passed the mark of one million subscribers! — the most for anything math-related (Vi Hart's channel is the only close competitor). MSRI has contributed both support and connections to some of the world's great mathematicians — I recommend the charming piece by Barry Mazur on right triangles, and the deep interview with John Conway — and with young mathematicians such as Holly Krieger, a postdoc in arithmetic dynamics at MIT who was at MSRI for a semester last year. I was amazed by the result on primes and iterated functions that she explains in her video. These and other treats can be found at http://www.numberphile.com/text_index.html.

Counting from Infinity: Yitang Zhang and the Twin Primes Conjecture. This is a film initiated and produced by MSRI for public TV, directed by George Csicsery. Zhang's personal story is truly amazing (the latest chapter, in case you've missed it, is his MacArthur award), and the theorem behind his fame is easy to explain to a non-mathematician, a great combination. Come see the world premiere at the Joint Math Meetings in San Antonio in January 2015!

Lyrical+Logical. Lyrical+Logical will be a series of prizes, awarded by MSRI, for children's books related to math appealing to kids of ages from 0 to 18. We are doing this in collaboration with the Children's Book Council (CBC), an organization to which some 70 publishers belong. Eager publishers have submitted over 80 new books and over 90 from backlists. Both the CBC and MSRI will activate their outreach channels to bring prominence to the books, both in bookstores and through librarians' lists. The books will also be distributed to underprivileged kids through collaborations with organizations such as First Book.

The National Math Festival. The National Math Festival, a first, will be held in Washington, DC, next April 16–18. It is a collaboration between MSRI and the Institute for Advanced Study. The first day will include a congressional briefing, a policy meeting on math education, and a gala dinner at the Library of Congress in support of basic research in mathematics and the sciences. Over the next two days there will be public lectures at some of the Smithsonian museums and the awarding of the first round of Lyrical+Logical prizes. NOVA, MoMath (the National Museum of Mathematics, located in NYC), Bridges, Guerilla Science, and other organizations will participate. It should be lots of fun — I hope you can join us for some of it!

New Geometric Methods in Number Theory and Automorphic Forms

(continued from page 1)

conjectures. In addition, the ideas and techniques recently introduced by Scholze, in particular his theory of perfectoid spaces, have already produced remarkable results, and their potential is only beginning to be realized. A common feature underlying many of these advances is the use of geometric techniques, often involving a link between geometry in characteristic 0 with that in characteristic $p > 0$. Many exciting new directions are emerging which are likely to occupy mathematicians working in the areas of number theory and automorphic forms for years to come.

In what follows, we will attempt to convey some of the basic mathematical ideas involved in the research activities of the New Geometric Methods in Number Theory and Automorphic Forms program. We have had to be selective and have omitted many interesting and vibrant parts of the picture, for example, Iwasawa theory and Euler systems, p-adic Hodge theory, the p-adic geometry of Shimura varieties, local models, Arakelov geometry, the relative trace formula and arithmetic relative trace formula, Rankin–Selberg methods and connections with ergodic theory and analytic number theory. Still, we hope that what we have included is representative of the spirit of the program.

Periods and L-values

An L-function is, roughly speaking, a function which shares the most striking features of Riemann's zeta function $\zeta(s) = \sum n^{-s}$ — namely, an analytic continuation, a functional equation, and a factorization over primes. Examples:

$$L(f,s) = \sum_{n=1}^{\infty} a_n n^{-s} \ , \quad L(K,s) = \sum_{n=1}^{\infty} b_n n^{-s} \ , \tag{1}$$

where $f(z) = \sum_{n=1}^{\infty} a_n q^n$, for $q = \exp(2\pi i z)$, is a modular form, and K is a finite extension of $\mathbb{Q}$ with b_n the number of ideals of K with norm n. Here it is more traditional to write $L(K,s) = \zeta_K(s)$ for the Dedekind zeta function of the field K. For example, if $K = \mathbb{Q}(i)$, then b_n is the number of ways of writing $n = a^2 + b^2$.

Let us examine some evaluations of some of these L-functions. Euler noted

$$\sum_{n=1}^{\infty} \frac{1}{n^{2k}} \in \mathbb{Q} \cdot \pi^{2k} \ , \tag{2}$$

and Hurwitz found the deeper result

$$\sum \frac{1}{(a+bi)^{4k}} \in \mathbb{Q}^{\times} \cdot \omega^{4k} \ , \tag{3}$$

where now the sum is over nonzero *Gaussian* integers $a + bi \in \mathbb{Z}[i] - \{0\}$, and $\omega = \int_0^1 \frac{dx}{\sqrt{1-x^4}}$.

These identities can be proved in similar ways by regarding them as specializations of meromorphic functions: for (2), we examine the meromorphic function $\sum_{n\in\mathbb{Z}} (n+z)^{-2k}$; this is highly constrained (being a meromorphic function which is, moreover, periodic under $z \mapsto z+1$) and this allows it to be easily computed as a rational function of $e^{2\pi i z}$. We then extract (2) by taking a limit as $z \to 0$. For (3), we may similarly consider the function $E_{4k}(z) = \sum (a+bz)^{-4k}$, summing over integers a and b not both zero, which is convergent so long as the imaginary part of z is not zero. This function is a weight k modular form, that is, in addition to being periodic under $z \mapsto z+1$, it satisfies the functional equation $E_{4k}(1/z) = z^{4k} E_{4k}(z)$, and this is a sufficiently strong constraint to almost pin it down.

These methods of proof generalize to the most powerful known method for obtaining results about L-functions, the method of periods of automorphic forms. In the example above, the series (3) arose as an evaluation of the modular form E_{4k} at the special point $z = i$; more generally, a vast body of work has suggested that many L-functions can be expressed as integrals of modular forms over special cycles.

For a first example, return to (1). Hecke established the analytic continuation of $L(f,s)$ by noting that one could express it in terms of the integral $\int_{y=0}^{\infty} f(iy) y^{s-1} dy$. To analytically continue, one splits into $y \in (0,1)$ and $y \in (1,\infty)$, and uses the functional equation of $f(z)$ to switch the problematic range $y \in (0,1)$ to $y \in (1,\infty)$.

In the general theory, an automorphic form is a certain function F on a locally symmetric space $[\mathbf{G}]$ attached to a reductive group $\mathbf{G}$. A "period" of F is then its integral

$$\int_{[\mathbf{H}]} F \tag{4}$$

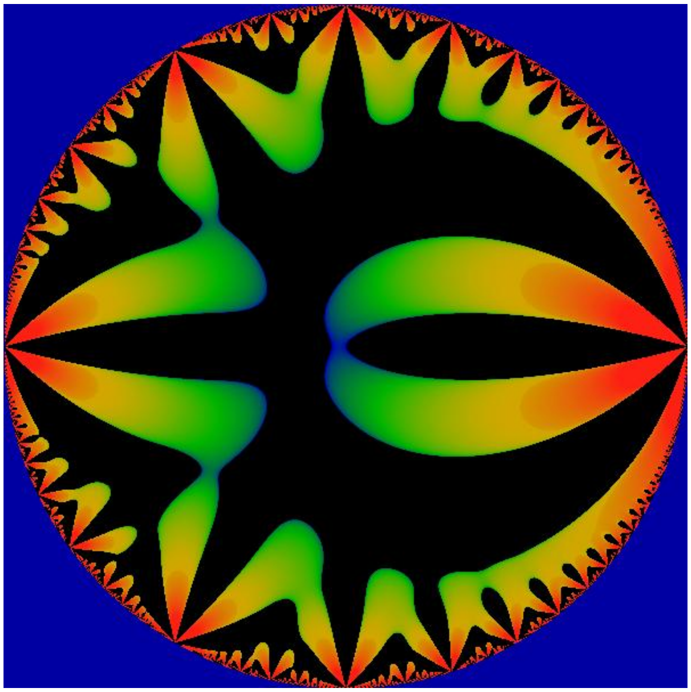
over the subspace attached to a subgroup $\mathbf{H} \subset \mathbf{G}$. In the example just discussed, $\mathbf{G} = \mathrm{GL}_2$ and $\mathbf{H}$ is the split torus of diagonal matrices.

For a more complicated example of such a period integral, we return to the second example of (1). Hecke also established the analytic continuation of $L(K,s)$ in the following way. If the degree of K over $\mathbb{Q}$ is equal to d, we may regard K as a subalgebra of $\mathrm{Mat}_d(\mathbb{Q})$, by choosing a $\mathbb{Q}$-basis for K and letting K act on itself via multiplication. The corresponding subgroup $K^{\times} \subset \mathrm{Mat}_d(\mathbb{Q})^{\times}$ defines a d-dimensional commutative subgroup $\mathbf{T}$ of GL_d. Hecke showed (albeit in different language) that $L(K,s)$ arises as a period over $\mathbf{H} = \mathbf{T}$ of a certain automorphic form on GL_d.

There is now a substantial body of work devoted to the problem of determining the choices of data $(\mathbf{G}, \mathbf{H}, F)$ for which the period (4) is related to an L-function, and if it is so related, how exactly to evaluate it. This includes the theory of Rankin–Selberg integrals, an active area in its own right.

In general, this circle of questions can be viewed as a "relative" Langlands program (that is, automorphic forms on $\mathbf{G}$ "relative to $\mathbf{H}$"), and there are many features parallel to the usual Langlands program, for example, a relative trace formula (Jacquet) and a relative L-group (Gaitsgory–Nadler, Sakellaridis–Venkatesh). While the theory is complete only in relatively few examples, there are precise conjectures (in particular the formalism of Ichino and

Ikeda) as well as some very deeply studied cases (the Gan–Gross–Prasad conjecture, with important cases recently established by W. Zhang) to guide the way. These ideas also interact with those around the Gross–Zagier formula (Yuan–Zhang–Zhang) as well as p-adic L-functions (Bertolini–Darmon–Prasanna), and have important applications in the study of algebraic cycles on Shimura varieties, height pairings, and special values of L-functions.



The imaginary part of $E_4(z)$ as a function of q in the unit disk.

The p-adic Langlands Program

To each prime number p, one can associate a multiplicative norm $|\,|_p$ on the field $\mathbb{Q}$ of rational numbers such that the more an integer n is divisible by p, the smaller $|n|_p$ is. The completion of $\mathbb{Q}$ for this norm is the field $\mathbb{Q}_p$ of p-adic numbers. One of the peculiarities of the p-adic world is that the integers no longer form a discrete set. Rather, they are dense in the ring $\mathbb{Z}_p = \{x \in \mathbb{Q}_p,\ |x|_p \leqslant 1\}$ of p-adic integers, and so the existence of a p-adically continuous function taking preassigned values at an infinite set of integers is far from automatic. For example, it is quite a miracle, noticed by Kubota and Leopoldt, that there exists a p-adic zeta function ζ_p, continuous on $\mathbb{Z}_p - \{1\}$, interpolating the values of the Riemann zeta function at negative integers: $\zeta_p(n) = (1-p^{-n})\zeta(n)$, for all integers $n \leqslant 0$ congruent to 1 modulo $p-1$.

Now, $\zeta(n)$, for $n \leqslant 0$, appears as the constant term of the Fourier expansion (q-expansion) of an Eisenstein series, which is a modular form of weight $1-n$. In the 1970s, Serre realized that this meant that one can construct, from these Eisenstein series, p-adic families of modular forms whose q-expansion varies continuously with the weight of the modular form. This led him to the notion of p-adic modular form and to a construction of the p-adic zeta function of totally real fields. Together with Deligne, he also used this circle of ideas to construct the p-adic Galois representations attached to forms of weight 1, by a p-adic limit process, from Galois representations attached to forms of weight $\geqslant 2$. In the case of weight $\geqslant 2$, such representations were previously constructed by Deligne using the étale cohomology of modular curves, but this construction does not work for weight 1.

Hida, in the 1980s, and Coleman–Mazur, in the 1990s, pushed this line of thought much farther, and Emerton, in this century, realized that many of their constructions could be better understood by looking at the cohomology with p-adic coefficients of the entire tower of modular curves. This led him to introduce "completed cohomology" groups, which come with a lot of extra equipment and whose study is one of the main goals of the p-adic Langlands program. In particular there are quite precise conjectures by Calegari and Emerton describing their structure. Modular curves arise as locally symmetric spaces associated to the group $GL_2(\mathbb{Q})$, but Emerton's construction makes sense for $GL_n(\mathbb{Q})$, or, indeed, for any reductive group G. For a general G, the tower of modular curves is replaced by a tower of higher dimensional locally symmetric spaces — now only differential manifolds and not necessarily algebraic varieties — and part of the extra structure alluded to above is an action of $G(\mathbb{Q}_\ell)$, for any prime number ℓ.

Now, for topological reasons, the action of $G(\mathbb{Q}_p)$ contains much more information than that of $G(\mathbb{Q}_\ell)$ for $\ell \neq p$, and the (still to be defined) p-adic local Langlands correspondence asks for a classification of representations of $G(\mathbb{Q}_p)$ arising in this way in terms of representations of the absolute Galois group $\mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$ of $\mathbb{Q}_p$. Such a classification is known only in the case $GL_2(\mathbb{Q}_p)$ thanks to the efforts of many people. That the question could have an answer was strongly advocated by Breuil, and a complete answer was recently obtained by Colmez–Dospinescu–Paskunas. The link between p-adic representations of $GL_2(\mathbb{Q}_p)$ and representations of $\mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$ is completely natural as it is given by a functor which transforms a representation of the first group into a representation of the second. The proof of this result rests upon ideas from p-adic Hodge theory, in particular, Fontaine's program of classification of Galois representations in terms of objects that are easier to handle. The result was used by Emerton, and by Kisin, to prove a great part of the Fontaine–Mazur conjecture for representations of dimension 2 of $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$. This conjecture is a far-reaching generalization of the Taniyama–Weil conjecture that was proven by Breuil–Conrad–Diamond–Taylor (1999) after the initial breakthrough of Wiles (1994) that led him to the proof of Fermat's last theorem. Breuil's quest was started by computations related to his work with Conrad, Diamond, and Taylor.

For groups other than GL_2, the situation is much less well understood, but there have been major breakthroughs in the last 2 years:

• Harris–Lan–Taylor–Thorne overcame one major difficulty in the construction of Galois representations attached to automorphic forms, namely that those that appear in cohomology are "self-dual." Scholze gave another proof of their result, using his theory of perfectoid spaces, and went on to make substantial progress on the conjectures of Calegari and Emerton mentioned above.

• Caraiani–Emerton–Gee–Geraghty–Paskunas–Shin constructed a candidate for the p-adic Langlands correspondence for GL_n using the completed cohomology, but the influence of the choices made in the construction is not clear for the moment.

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• Scholze defined a functor that attaches to a p-adic representation of $GL_n(\mathbb{Q}_p)$ a representation of $Gal(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$ and showed that this functor gives "the" right answer for representations coming from completed cohomology.

New Geometry: Perfectoid Spaces

It is hard to overstate the impact on recent arithmetic geometry of Peter Scholze's theory of *perfectoid spaces*, which builds on work of Fontaine–Wintenberger, Faltings, Kedlaya and many others. Let us sketch out the themes of this theory and its applications.

We have already mentioned the ring $\mathbb{Z}_p$ of p-adic integers. This is the set of formal power series

$$a_0 + a_1 p + a_2 p^2 + \dots ,$$

where each coefficient a_i lies in $\{0, 1, \dots, p-1\}$.

When adding or multiplying two elements of $\mathbb{Z}_p$, one must perform "carries" when one encounters a coefficient outside this range. If one ignores the carries, one gets a different ring $\mathbb{F}_p[[t]]$, the ring of formal power series over $\mathbb{F}_p$. Both $\mathbb{F}_p[[t]]$ and $\mathbb{Z}_p$ are discrete valuation rings with residue field $\mathbb{F}_p$, but the structure of $\mathbb{F}_p[[t]]$ is arguably much simpler, roughly because t is a free indeterminate, whereas $p = 1 + 1 + \cdots + 1$. Note that the same sum equals 0 in $\mathbb{F}_p[[t]]$, because we did not bother carrying, and therefore this ring has *characteristic* p.

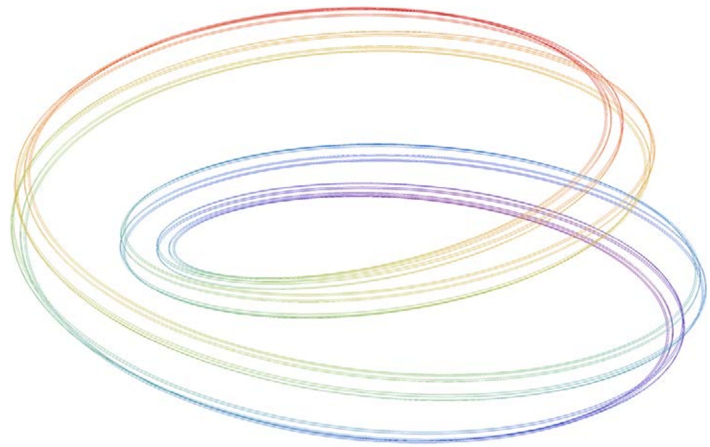
The work of Fontaine–Wintenberger (1979) provided a link between the absolute Galois groups of $\mathbb{Q}_p$ and $\mathbb{F}_p((t))$ (this is the fraction field of $\mathbb{F}_p[[t]]$). This link only appears, however, once you pass from $\mathbb{Q}_p$ to a larger field such as

$$K = \mathbb{Q}_p(p^{1/p^\infty}) = \bigcup_{n=1}^{\infty} \mathbb{Q}_p(p^{\frac{1}{p^n}})$$

(or rather its p-adic completion), which is wildly ramified over $\mathbb{Q}_p$. This field has the properties that (a) its value group is not discrete, and (b) the pth power map $\mathcal{O}_K/p \to \mathcal{O}_K/p$ is surjective. Here $\mathcal{O}_K$ is the ring of elements of absolute value $\leqslant 1$. Scholze gives the name *perfectoid field* to fields K of this type. The theorem of Fontaine–Wintenberger is that $Gal(\overline{K}/K)$ is isomorphic to the Galois group of a field in characteristic p, which in our example is $\mathbb{F}_p((t))$.

In the 2010s, Scholze introduced a class of geometric objects called *perfectoid spaces*, which satisfy analogues of properties (a) and (b) above. Perfectoid fields are the points in this theory. A typical example of a perfectoid space is the set of sequences $(x_0, x_1, \dots)$ of elements belonging to (say) $\mathbb{C}_p = \hat{\overline{\mathbb{Q}}}_p$, the completion of an algebraic closure of $\mathbb{Q}_p$, which satisfy $x_n^p = x_{n-1}$ for all $n \geqslant 1$. Note that if $\mathbb{C}_p$ is replaced with $\mathbb{C}$ in this formulation, say with the further restriction that the x_n's lie in the unit circle, the resulting space is a solenoid; it is nothing at all like a manifold — in fact it is connected, but not path-connected. This should give a sense of how exotic perfectoid spaces are.

Scholze proved that there is an equivalence $X \mapsto X^\flat$ between perfectoid spaces in characteristics 0 and p, called *tilting*. The initial application was to prove Deligne's long-standing weight-monodromy conjecture, concerning the cohomology of varieties over a local field, by transferring it into characteristic p, where it had been previously known.



A solenoid formed from the circle S^1; the simplest perfectoid spaces are p-adic analogues.

There have been a number of striking applications of perfectoid spaces since then, including the previously mentioned construction of Galois representations associated to regular algebraic automorphic representations which aren't necessarily self-dual.

The most recent application is the subject of Scholze's Chancellor's Lectures at UC Berkeley. This is an attempt to extend the work of Drinfeld, L. Lafforgue and V. Lafforgue on the Langlands correspondence over function fields (such as $\mathbb{F}_p(t)$) to a characteristic 0 (but local) setting. Very roughly speaking, the idea is to use the notion of tilting to make possible constructions in characteristic 0 which previously only made sense in characteristic p. The goal is to construct a moduli space of mixed-characteristic "shtukas" whose cohomology is conjectured to exhibit the Langlands correspondence for an arbitrary reductive group G over $\mathbb{Q}_p$.

Bridges to Geometric Representation Theory

Much of the research activity of this current program has an essential geometric aspect, although the notion of "geometry" must be understood very broadly to include arithmetic geometry, Arakelov geometry, rigid and adic geometry, perfectoid spaces, etc, in addition to more classical algebraic and differential geometry. The emphasis on geometric aspects also provides a bridge with the concurrent MSRI program on Geometric Representation Theory. Indeed, a large part of geometric representation theory is motivated by the desire to geometrize many constructions in number theory and automorphic forms. In return, insights and techniques from geometric representation theory have led to solutions of some longstanding problems in arithmetic, as exemplified by Ngô's proof of the fundamental lemma. The cross pollination between these two areas will continue to be important and mutually beneficial.

Focus on the Scientist:
Peter Scholze

Peter Scholze is a member of the MSRI program on New Geometric Methods in Number Theory and Automorphic Forms, and also the UC Berkeley Chancellor's Professor. He is a professor at Universität Bonn.

While still a graduate student at Bonn, Peter made a spectacular breakthrough in the field of arithmetic geometry with his invention of perfectoid spaces. Around the same time, he devised a new proof of the local Langlands conjectures for general linear groups over p-adic fields, which greatly simplified prior work of Michael Harris and Richard Taylor. He advanced from his Ph.D. directly to a full professorship at Bonn, becoming at the age of 24 the youngest professor in Germany to achieve this rank.

Since their invention, perfectoid spaces have been the focus of study groups and workshops around the world, including an MSRI Hot Topics workshop in February 2014. They were introduced to study algebraic varieties over a p-adic field. In brief, a perfectoid space is what lies at the top of an infinite tower of varieties, each one mapping to the next in a way which adjoins pth roots of functions modulo p. Even if the variety at the bottom of the tower is smooth, the perfectoid space on top will look something like a fractal. One virtue of perfectoid spaces is that they admit a "tilting" operation, which faithfully transforms them into objects in characteristic p. Using this operation, Peter proved Deligne's weight-monodromy conjecture for a large class of varieties, by reducing to the case of characteristic p where it had already been known.

Two subsequent projects used perfectoid spaces to achieve stunning results which hardly anyone could have anticipated. In the first, Peter developed Hodge theory for varieties in a p-adic context, thereby proving a conjecture posed by John Tate in 1967. In the second, he established a deep connection between the geometry of so-called "arithmetic manifolds" (for example, the quotient of hyperbolic 3-space by an arithmetic subgroup of $PSL_2(\mathbb{C})$), and representations of the Galois group of a number field. Currently in his capacity as Chancellor's Professor, Peter is giving a series of lectures at UC Berkeley outlining a new theory of "mixed characteristic shtukas" which may prove transformative in the study of the local Langlands program.

Peter Scholze

Peter's mathematical style resembles Grothendieck's in that he builds nimble, well-oiled theories which work in maximum generality, with a rare intuition for studying the "right" classes of objects. This is remarkable enough, but Peter doesn't just build machines; he uses them to knock down one towering wall after another, laying bare a beautiful mathematical landscape for us all to enjoy.

— Jared Weinstein

Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the Scientific Advisory Committee with a copy to proposals@msri.org. For detailed information, please see the website www.msri.org.

Thematic Programs

Letters of intent and proposals for semester or year long programs at the Mathematical Sciences Research Institute (MSRI) are considered in the fall and winter each year, and should be submitted preferably by **October 15** or **December 15**. Organizers are advised that a lead time of several years is required, and are encouraged to submit a letter of intent prior to preparing a pre-proposal. For complete details see http://tinyurl.com/msri-progprop.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals for such workshops should be submitted by **October 15** or **December 15**. See http://tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes four 2-week long summer graduate workshops, most of which are held at MSRI. To be considered for the summer of year n, proposals should be submitted by **October 15** or **December 15** of year $n-2$. See http://tinyurl.com/msri-sgs.

Geometric Representation Theory

David Ben-Zvi and Kevin McGerty†

What is Geometric Representation Theory?

Representation theory is the study of the basic symmetries of mathematics and physics. The primary aim of the subject is to understand concrete linear models for abstract symmetry groups. A signature triumph of the past century is our understanding of the representation theory and harmonic analysis of compact Lie groups. Among its cornerstones are the Cartan–Killing classification, the uniform construction by Borel–Weil–Bott of all representations in the cohomology of line bundles on flag varieties, and the nonabelian Fourier theory provided by the Peter–Weyl theorem. Thus we know every compact Lie group, every way in which it could appear concretely as a matrix group, and how to use its appearance as a symmetry group to describe function spaces. The ideas and results of this subject are the basic input to diverse areas from number theory to quantum field theory.

Though mysteries still remain, the theory of compact Lie groups yields a paradigm for what we would like to achieve with other symmetry groups. In broad strokes, a large part of modern representation theory depends on three variables. First we fix a reductive group, roughly specifying the "type" of group to consider — for example, the groups GL_n of all invertible matrices, O_n of orthogonal matrices and Sp_n of symplectic matrices. Next we choose a field or ring of definition k, from which to take the entries of our matrices. This leads to finite groups of Lie type ($k = \mathbb{F}_q$), p-adic groups ($k = \mathbb{Q}_p$ or a finite extension), loop groups ($k = \mathbb{C}((t))$ or $\mathbb{F}_q((t))$) and adèle groups ($k = \mathbb{A}$, the ring of adèles of a number field such as $\mathbb{Q}$ or of the function field of a curve over a finite field). Finally, we may consider representations of the group on vector spaces defined over another field F, resulting in complex, modular, and p-adic representation theories (among others). In contrast with the story for compact Lie groups, our understanding of these varied representation theories is still very coarse, though the last decades have witnessed breathtaking advances.

Geometric representation theory seeks to understand these groups and their representations as a consequence of more subtle but fundamental structures. Each reductive group G possesses a cast of closely related interesting (and often singular) spaces. One then realizes representations of G over a field F as cohomology groups with coefficients given by sheaves of F-vector spaces. Among these spaces and resulting representation theories are the flag manifolds (Lie groups and algebras), Deligne–Lusztig varieties (finite groups of Lie type), Springer fibers (Weyl groups and Hecke algebras), quiver varieties (Kac–Moody algebras and quantum groups), affine Grassmannians and general moduli spaces of bundles on curves (algebraic groups and loop groups), buildings (p-adic groups), Drinfeld modular varieties, and Shimura varieties (adèle groups over function fields and number fields, respectively).

We will now explore some of the themes pursued by members in the Geometric Representation Theory (GRT) program (whom we marked with an asterisk). We apologize in advance that in such a brief survey it has been impossible to be comprehensive either in topics or attributions.

$\mathcal{D}$-modules and Quantization

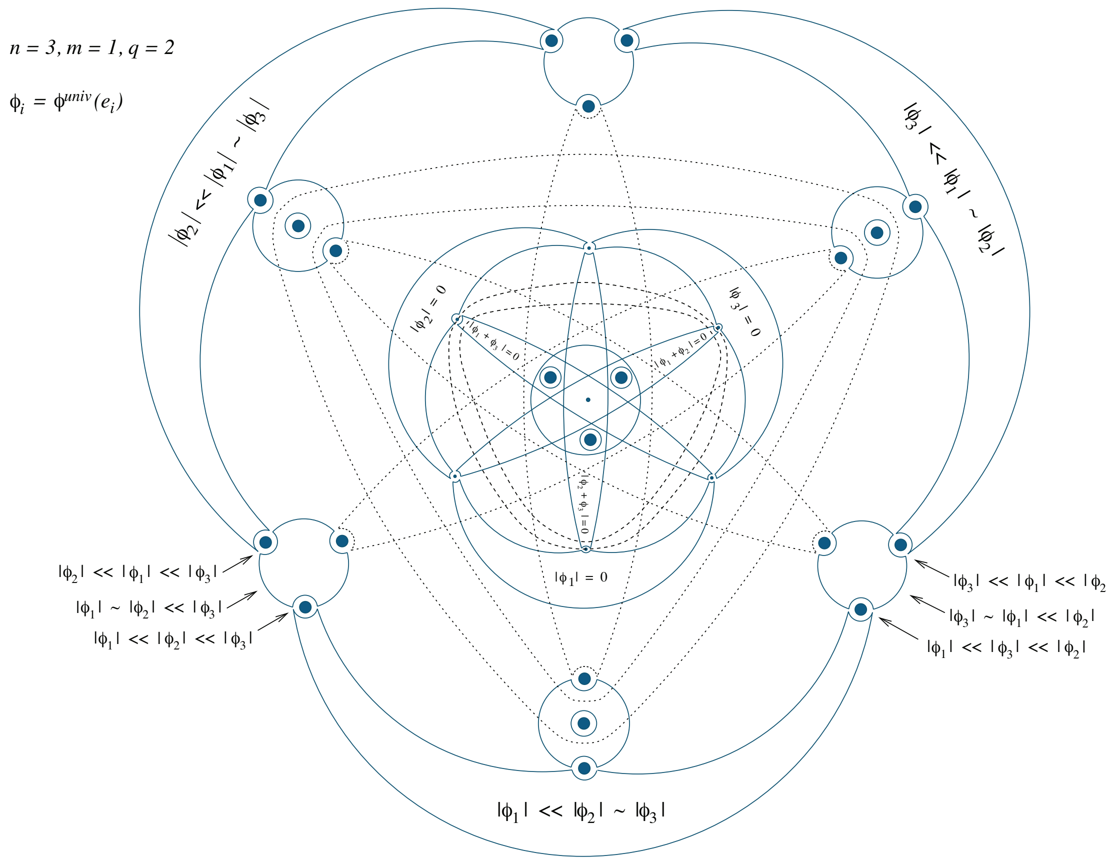
One of the groundbreaking successes in geometric representation theory is the vast generalization of the Borel–Weil–Bott theorem provided by Beilinson and Bernstein*'s uniform construction of all representations of (reductive, not necessarily compact) Lie groups and algebras using $\mathcal{D}$-modules (algebraic systems of differential equations) on flag varieties. By passing from differential equations to the monodromy properties of their solutions, the Riemann–Hilbert correspondence relates these $\mathcal{D}$-modules to purely topological objects, perverse sheaves, which in turn benefit from the powerful toolkit of Hodge theory. Among the consequences of this profound link between representation theory and topology are the celebrated Kazhdan–Lusztig conjectures on the characters of representations of reductive Lie algebras.

The insights of these results have been brought to bear on several other kinds of representations. In the setting of Lie algebras in positive characteristic, Bezrukavnikov and Mirkovic* solved a series of influential conjectures of Lusztig by combining a positive characteristic version of Beilinson–Bernstein* theory, categorical braid group actions showing the representation theory is "independent of p" for large p, and a geometric Langlands duality describing the characteristic zero version. There is now a concerted push (by GRT members Huyghe* and Strauch* among others) to develop a new rigid analytic Beilinson-Bernstein* theory to bear on an important new frontier, the p-adic representation theory of p-adic groups.

In the setting of complex geometry, the study of $\mathcal{D}$-modules on the flag manifold can be viewed as a special case of the study of deformation quantization of holomorphic symplectic resolutions (in this case, the cotangent to the flag manifold is the Springer resolution of the cone of nilpotent elements in a Lie algebra). An emerging paradigm (inspired in part by supersymmetric quantum field theory, and represented in the GRT program by Dodd*, McGerty*, and Nevins*) attaches a "representation theory" (for example that of rational Cherednik algebras) to any symplectic resolution (for example, Hilbert schemes of points).

Quantization, as seen through the lens of differential and difference equations and their monodromy, gives rise to geometric realizations of many important algebras. A fundamental result in this direction is the Drinfeld–Kohno theorem, which in effect constructs quantum groups using the monodromy of differential equations built out of the representation theory of loop groups. The work of GRT members Jordan*, Toledano Laredo*, and Yang* is concerned with generalizations and applications of this bridge in new

†With contributions by Pramod Achar, Thomas Haines, Ivan Mirkovic, Matthias Strauch, Valerio Toledano Laredo, David Treumann, and Zhiwei Yun. The names of members of the GRT program have been marked throughout the article with an asterisk.



The tubular neighborhood of a singular point on a Shimura variety with $\Gamma(\mathfrak{p})$-level structure. (Figure from Matthias Strauch.)

settings, including double affine Hecke algebras, quantum affine algebras and elliptic quantum groups.

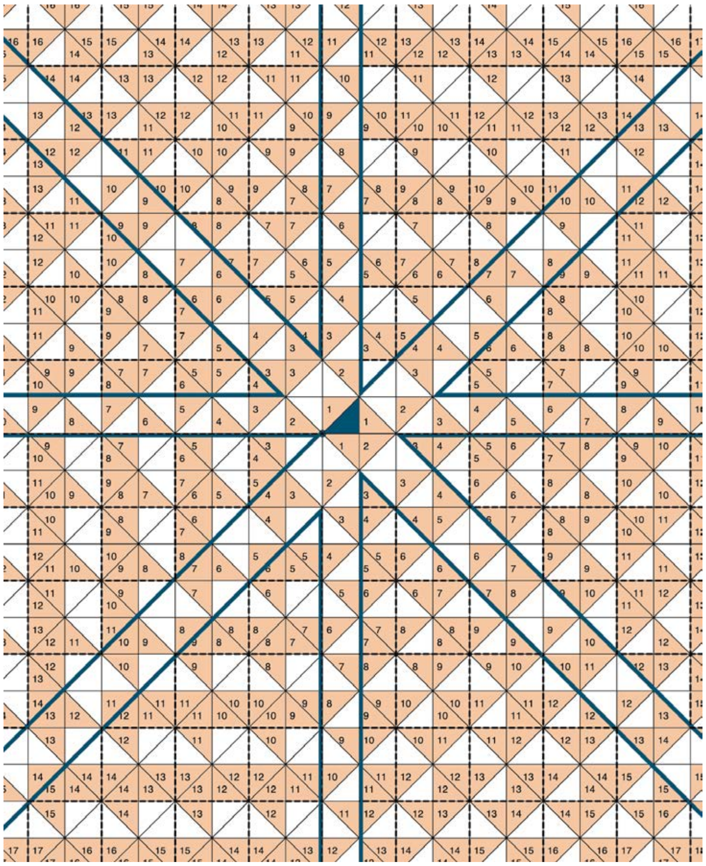
Modular Representations and Sheaves

A fundamental geometric theory of representations is provided by the form of the geometric Satake correspondence established by Mirkovic* and Vilonen*. This asserts that the algebraic representations of a reductive group with coefficients in any ring k can be realized as the cohomologies of perverse sheaves with k coefficients on a fixed (complex) space, the affine Grassmannian associated to the dual group. The study of sheaves with positive characteristic coefficients (or "modular sheaves" as they have come to be called) is, however, relatively uncharted geometric terrain — the familiar tools of Hodge theory such as the decomposition theorem fail here, as do the lucky vanishings of odd cohomology groups in key examples which allowed one to avoid tricky cancellations. The representation theory is equally uncharted: Williamson recently showed that the long expected patterns (Lusztig's conjecture) fail in characteristics of intermediate size. Enter parity sheaves. This notion, introduced by Juteau*–Mautner*–Williamson in 2009, is at once elementary and deeply insightful. They are defined by enforcing cohomological vanishing in odd (or even) degree, and are in turn lucky in that a form of the decomposition theorem holds in key examples. Through a flurry of activity, involving also GRT members Achar* and Fiebig*, parity sheaves have emerged as important tools for representation theoretic problems, and are the subject of an MSRI working group this semester.

The Geometric Langlands Correspondence

Perhaps the greatest challenge in representation theory of reductive groups is the organization of representations according to the visionary roadmap provided by the Langlands program. This is a vast generalization of class field theory, which identifies representations of multiplicative groups over local and global fields with characters of Galois groups. The Langlands philosophy proposes a Fourier theory for reductive groups, parametrizing representations of reductive groups in any of the above contexts in terms of "dual" arithmetic or topological data and using these as the basis for a general harmonic analysis. Through the work of many people, foremost among them Beilinson, Drinfeld, and Laumon*, a geometric version of this program now exists in the setting of curves over finite fields and the complex numbers, which is currently the subject of intense investigation. A crucial feature of the geometric Langlands program is that it provides a "lift" of the classical program to a categorical level, allowing a deeper understanding of the relation between the "representation theoretic" and "Galois" sides and resulting in a new kind of "categorical harmonic analysis" which clarifies many aspects of classical representation theory. The theme of categorification, ubiquitous in contemporary representation theory but tracing back to Grothendieck's categorification of functions by ℓ-adic sheaves, typically turns numbers into the dimensions of vector spaces (whence integrality properties) and vector spaces into K-groups of categories (whence canonical bases). This theme is fundamental to the work of GRT members Cautis*, Dancso*, Lauda*, Stroppel*, and Vazirani*.

GRT *(continued from previous page)*



The dimensions of affine Deligne–Lusztig varieties $X_w(b)$ in the affine flag variety for GSP(4) attached to a "basic" element $b = \tau$. (Figure from Ulrich Görtz.)

In the complex setting, Beilinson and Drinfeld uncovered several deep connections between the geometric Langlands program and mathematical physics. A central object is the Hitchin fibration on the moduli of Higgs bundles on a Riemann surface, an integrable system with origins in gauge theory which is a version of the characteristic polynomial map on matrices parametrized by the surface. The connection with physics was advanced significantly by the work of Kapustin and Witten, who tied the geometric Langlands program to a nonabelian gauge theory generalization of electric-magnetic duality just as the Langlands program generalizes class field theory. This revealed many additional layers of structure that can be encapsulated in the form of topological quantum field theory, itself an area of very active investigation. The perspective on representation theory coming from physics is central to the work of GRT members Ben-Zvi*, Dobrovolska*, Frenkel*, Gunningham*, Jordan*, and Nadler*.

Another level of sophistication was recently added to geometric representation theory through the development of derived algebraic geometry by Lurie, Toën, and Vezzosi. This language is necessary to capture the subtleties of functional and harmonic analysis in the categorical setting, and hence for the correct formulation of Langlands duality in the categorical setting. It is the basis of great recent progress by Gaitsgory, Arinkin, Drinfeld, and coauthors including program members Raskin* and Rozenblyum*. The new perspective combining derived geometry with quantum field theory has already permitted a deeper understanding of geometric Langlands duality, and indeed one might optimistically say that a proof of this duality is now within sight.

From geometry to arithmetic

One of the most exciting developments driving the GRT program is the application of methods from geometric representation theory to solve problems originating in number theory. A spectacular example is the proof of the fundamental lemma by Ngô*. This is an identity of orbital integrals for p-adic groups conjectured by Langlands and Shelstad, which plays a crucial role in the global theory of automorphic forms. The work of Waldspurger and the "logical representation theory" of Cluckers, Hales*, and Loeser allow one to deduce this identity from its analogue for local fields of characteristic p (for almost all p). Ngô* then proves the result using the global geometry of curves over finite fields. The crucial step is a geometric interpretation of the problem in terms of the cohomology of the Hitchin fibration, exhibiting a close tie with the geometric Langlands program. These ideas have been very influential and feature in recent work of GRT members Bouthier*, Frenkel*, Gordon*, Laumon*, Ngô*, and Yun*.

Several current developments are eroding the divide between the geometric setting of function fields and the arithmetic problems of p-adic fields and number fields. Using sophisticated methods from the geometric Langlands correspondence, Yun* was able to construct motives over number fields with exceptional motivic Galois groups and to solve new cases of the celebrated inverse Galois problem over the rational numbers.

Other developments involve Shimura varieties, generalizations of the moduli space of elliptic curves whose cohomology provides a natural setting in which to realize cases of the Langlands correspondence. Special emphasis has been devoted to the study of the reduction of these varieties modulo a prime number, and particularly interesting is the case when these special fibers have singularities. Much recent progress has resulted from exploiting connections between the rich geometry of these singularities and that of loop groups (affine Grassmannians and flag varieties). One example is work on Kottwitz' conjecture relating the action of Frobenius on cohomology of special fibers with the center of the Hecke algebra, recently solved by Pappas and Zhu* by combining loop and p-adic directions (following cases proved by Haines*, Ngô*, and Rostami*). Another example is work of Hamacher* on the Newton stratification in special fibers of Shimura varieties translating techniques used by Görtz, Haines*, Kottwitz, Reuman, and Viehmann* to study affine Deligne–Lusztig varieties in affine flag varieties. The connection is becoming even deeper: recently Zhu* constructed an object that has long been sought, namely a p-adic avatar of the affine Grassmannian, and used it to prove a geometric Satake equivalence for local fields of mixed characteristics. This opens up the possibility of proving results about mixed characteristic local fields directly using geometry over a finite field.

Finally, a thrilling vista from MSRI is Scholze's lecture series, presenting a new p-adic geometry including a p-adic version of Drinfeld modular varieties for function fields. This raises the possibility of a purely geometric understanding of the local Langlands correspondence in parallel to the seminal results of Drinfeld, L. Lafforgue, and V. Lafforgue in the function field setting.

NAMC at MAA MathFest

Brandy Wiegers

The National Association of Math Circles (NAMC) participated in the Mathematical Association of America (MAA) MathFest for the sixth time. The goal was to introduce the broader mathematical community to the resources that are available to those who would like to run Math Circle programs at their institutions. During NAMC's first participation at the MAA MathFest six years ago, very few visitors knew what a Math Circle was, but this year nearly everyone had heard of Math Circles and was excited for the many resources that NAMC was able to share.

NAMC and the "Special Interest Group of the MAA on Circles" (SIGMAA–MCST) jointly sponsored a booth at the event in Portland where MathFest participants were encouraged to take a break and play. James Tanton, Phil Yasskin, Amanda Serenevy, Dave Auckly, Japheth Wood, and Brandy Wiegers all shared hands-on engaging Math Circle lessons. Dice were thrown, balloons were tied together, and tea cups were twisted. Other Math Circle leaders answered questions and shared many ideas for unique activities. NAMC resources are available online at mathcircles.org.

Focus on the Scientist:
Joseph Bernstein

Joseph Bernstein is a Clay Senior Scholar in the Geometric Representation Theory program. He is a luminary in representation theory and his far-reaching ideas have shaped numerous branches of the subject. His contributions to mathematics are myriad and profound, and it is impossible to fully capture the extent of his achievements in a brief profile.

Joseph introduced many of the most important objects of study in representation theory as well as powerful techniques for understanding them. For example, a central theorem in geometric representation theory is the 1981 Beilinson–Bernstein localization theorem, which describes representations of a reductive Lie algebra as (twisted) $\mathcal{D}$-modules on the flag variety. The theory of $\mathcal{D}$-modules itself was initiated earlier by Joseph himself. In the late 1960s, he introduced the notion of holonomicity of $\mathcal{D}$-modules and the Bernstein–Sato polynomials to resolve a conjecture of Gelfand regarding analytic continuation of certain functions.

An immediate consequence of the localization theorem is the Kazhdan–Lusztig conjecture, relating the structure of category O to the topology of the flag variety. Relatedly, in 1982, Beilinson, Bernstein and Deligne introduced the concept of perverse sheaves (a topological analogue of $\mathcal{D}$-modules) and proved the BBD decomposition theorem, which has since become a fundamental tool in geometric representation theory. Category O itself was introduced in the 1970s by Bernstein, Gelfand, and Gelfand in a series of papers which also constructed the famed BGG resolution. Today, category O and its numerous analogues are a central object of study in representation theory.

Joseph has made similarly important contributions to the representation theory of p-adic groups, including the Bernstein decomposition of the category of smooth complex representations, the Bernstein center, and Bernstein's presentation of affine Hecke algebras. He proved the second adjointness theorem, which gives an unexpected relationship between parabolic induction and Jacquet functors. More recently, over the past decade, Bernstein's work with Reznikov introduced new ideas from representation theory to the study of analytic questions in the theory of automorphic forms.

Joseph Bernstein

In addition to his contributions in research, Joseph is also a legendary advisor. Many of his students have become thought leaders in the field and are a testament to his remarkable abilities as a teacher.

Joseph received his Ph.D. under I.M. Gelfand at Moscow State University in 1972. He was a professor at Harvard from 1983 until 1994, when he moved to Tel Aviv University, where he is currently professor emeritus. Joseph is a member of the Israel Academy of Sciences and Humanities, and, in 2004, he was awarded the Israel Prize and was elected to the National Academy of Sciences.

— Nick Rozenblyum

Forthcoming Workshops & Programs

Workshops

October 29–November 1, 2014: *Breaking the Neural Code*, organized by Larry Abbott, Ingrid Daubechies, Michael Jordan, and Liam Paninski (Lead)

November 1, 2014: *Bay Area Differential Geometry Seminar (BADGS) Fall 2014*, organized by David Bao, Joel Hass, David Hoffman (Lead), Rafe Mazzeo, and Richard Montgomery

November 17–21, 2014: *Categorical Structures in Harmonic Analysis*, organized by Thomas Haines, Florian Herzig, and David Nadler (Lead)

December 1–5, 2014: *Automorphic Forms, Shimura Varieties, Galois Representations and L-functions*, organized by Pierre Colmez (Lead), Stephen Kudla, Elena Mantovan, Ariane Mézard, and Richard Taylor

January 15–16, 2015: *Connections for Women: Dynamics on Moduli Spaces of Geometric Structures*, organized by Virginie Charette, Fanny Kassel (Lead), Karin Melnick, and Anna Wienhard

January 20–23, 2015: *Introductory Workshop: Dynamics on Moduli Spaces of Geometric Structures*, organized by Richard Canary, William Goldman (Lead), Ursula Hamenstädt, and Alessandra Iozzi

January 29–30, 2015: *Connections for Women: Geometric and Arithmetic Aspects of Homogeneous Dynamics*, organized by Elon Lindenstrauss and Hee Oh

February 2–6, 2015: *Introductory Workshop: Geometric and Arithmetic Aspects of Homogeneous Dynamics*, organized by Manfred Einsiedler, Jean-François Quint (Lead), and Barbara Schapira

March 9–13, 2015: *Hot Topics: Kadison–Singer, Interlacing Polynomials, and Beyond*, organized by Sorin Popa, Daniel Spielman (Lead), Nikhil Srivastava, and Cynthia Vinzant

March 18–20, 2015: *Critical Issues in Mathematics Education 2015: Developmental Mathematics: For Whom? Toward What Ends?*, organized by Duane Cooper, Mark Hoover, Robert Megginson (Lead), Richard Sgarlotti, and Katherine Stevenson

April 13–17, 2015: *Dynamics on Moduli Spaces*, organized by Markus Burger, David Dumas, Olivier Guichard, François Labourie, and Anna Wienhard

May 11–15, 2015: *Advances in Homogeneous Dynamics*, organized by Dmitry Kleinbock (Lead), Hee Oh, Alireza Salehi Golsefidy, and Ralf Spatzier

Programs

January 12–May 22, 2015: *Dynamics on Moduli Spaces of Geometric Structures*, organized by Richard Canary, William Goldman, François Labourie, Howard Masur (Lead), and Anna Wienhard

January 19–May 29, 2015: *Geometric and Arithmetic Aspects of Homogeneous Dynamics*, organized by Dmitry Kleinbock (Lead), Elon Lindenstrauss, Hee Oh, Jean-François Quint, and Alireza Salehi Golsefidy

August 17–December 18, 2015: *New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems*, organized by Kay Kirkpatrick, Yvan Martel, Jonathan Mattingly, Andrea Nahmod, Pierre Raphael, Luc Rey-Bellet, Gigliola Staffilani (Lead), and Daniel Tataru

January 11–May 20, 2016: *Differential Geometry*, organized by Tobias Colding, Simon Donaldson, John Lott, Natasa Sesum, Gang Tian, and Jeff Viaclovsky (Lead)

For more information about any of these workshops as well as a full list of all upcoming workshops and programs, please see www.msri.org/scientific.

Sam Gunningham

Viterbi Postdoc

Sam Gunningham is this fall's Viterbi Endowed Postdoctoral Fellow and is a member of the program on Geometric Representation Theory. Sam did his undergraduate studies at Cambridge University in the United Kingdom. In the fall of 2008, he came to Northwestern University for his doctoral studies, which he completed under the supervision of David Nadler. Since Fall 2013 he has been a Bing Instructor in Mathematics at the University of Texas in Austin.

In his research, Sam has applied extended topological field theory to give an elegant solution to a problem from enumerative geometry, counting spin covers of a Riemann surface. He is currently working on a categorification of the Harish-Chandra isomorphism and its applications to cohomology of character varieties of surface groups.

The Viterbi Endowed Postdoctoral Scholarship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access (CDMA) based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.

Strauch Postdoc

Jasmin Matz, a member of the New Geometric Methods in Number Theory and Automorphic Forms program, is the Fall 2014 Strauch Endowed Postdoctoral Scholar.

Jasmin Matz

Jasmin did her undergraduate studies in Germany at Universität Duisburg-Essen and obtained her Diplom in 2008. She was awarded a prize for the best diploma thesis in mathematics in 2008. She obtained her Ph.D. in 2011 under the supervision of Tobias Finis at Heinrich-Heine Universität Düsseldorf (Germany).

Jasmin works on the analytic theory of automorphic forms and, in particular, applications of the Arthur–Selberg trace formula. Before starting her position at MSRI, Jasmin was a postdoctoral fellow at the Max-Planck Institut für Mathematik, Bonn (Germany), a Golda Meir postdoctoral fellow at the Hebrew University of Jerusalem (Israel) and a Scientific Assistant in Universität Bonn (Germany).

The Strauch Fellowship is funded by a generous annual gift from Roger Strauch, Chairman of The Roda Group. He is a member of the Engineering Dean's College Advisory Boards of UC Berkeley and Cornell University, and is also currently the chair of MSRI's Board of Trustees, on which he has served for more than 15 years.

Berlekamp Postdoc

Bao Viet Le Hung, a member of the New Geometric Methods in Number Theory and Automorphic Forms program, is the Fall 2014 Berlekamp Endowed Postdoctoral Scholar.

Bao Viet Le Hung

Bao Viet completed his undergraduate studies at University of Cambridge. In 2009, he was awarded a Pierce Fellowship to study with Richard Taylor at Harvard University. In his thesis work, completed in 2014, he showed the modularity of elliptic curves over real quadratic fields up to finite exceptions, and with Nuno Freitas and Samir Siksek, removed these exceptions later on. Such a definitive result is a huge and astonishing achievement.

With Ana Caraiani, he computed the image of the complex conjugation in Galois representation associated to regular algebraic automorphic representations for GL_n over a totally real field. Their elegant method provided yet another surprise for experts.

The Berlekamp Postdoctoral Fellowship was established in 2014 by a group of his friends, colleagues, and former students whose lives have been touched by Dr. Berlekamp in many ways. He is well known for his algorithms in coding theory and has made important contributions to game theory. He is also known for his love of mathematical puzzles.

Endowed and Named Positions for Fall 2014

MSRI is grateful for the generous support that comes from endowments and annual gifts that support faculty and postdoc members of its programs each semester.

Eisenbud and Simons Professors

Geometric Representation Theory
David Ben-Zvi, University of Texas, Austin
Joseph Bernstein, Tel Aviv University
Thomas Hales, University of Pittsburgh
Florian Herzig, University of Toronto
Kevin McGerty, Oxford University
Ivan Mirkovic, University of Massachusetts
Ngô Bảo Châu, University of Chicago
Marie-France Vigneras, Institut de Mathématiques de Jussieu

New Geometric Methods in Number Theory and Automorphic Forms
Christophe Breuil, Université Paris-Sud
Pierre Colmez, Université Pierre et Marie Curie
Wee Teck Gan, National University of Singapore
Stephen Kudla, University of Toronto
Elena Mantovan, Caltech
Ariane Mezard, Université Pierre et Marie Curie
Michael Rapoport, Mathematisches Institut Universität Bonn
Akshay Venkatesh, Stanford University

Named Postdoctoral Fellows

Geometric Representation Theory
Cha–Chern: Yaping Yang, Northeastern University
Viterbi: Sam Gunningham, Northwestern University

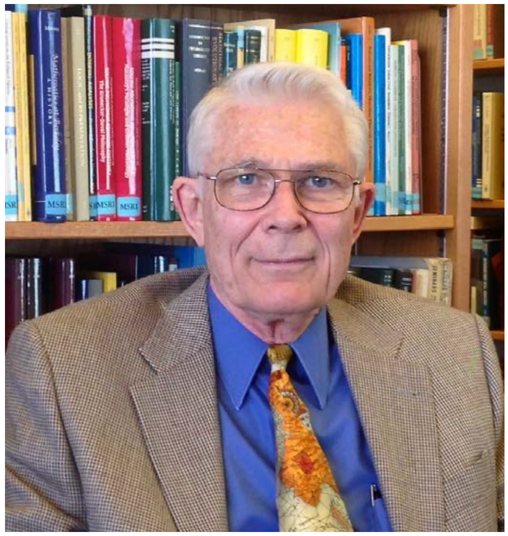
New Geometric Methods in Number Theory and Automorphic Forms
Berlekamp: Bao Viet Le Hung, Harvard University
Strauch: Jasmin Matz, University of Bonn

Gamelin Postdoctoral Fellowship

New Fellowship with a Focus on Education

Heike Friedman

MSRI provides unique opportunities for postdoctoral fellows by bringing them together with leading researchers in the field in an environment that promotes creativity and the effective interchange of ideas and techniques. Thirty-five postdoctoral fellows spend a semester or more at MSRI every year, and six of them are supported by privately funded fellowships.



Ted Gamelin in the MSRI library

The newest of our named fellowships was created just a few months ago by Dr. Ted Gamelin with a generous gift to our Endowment Fund. The Gamelin Fellowship will have an emphasis on the important role that research mathematicians play in the discourse of K-12 education. Therefore it will be awarded during MSRI's spring programs to ensure the postdoc will be at MSRI during each year's workshop on Critical Issues in Mathematics Education (CIME).

"Over the past fifteen years or so I have been involved in the California textbook approval process," says Dr. Gamelin. "First as a math specialist sitting on approval panels and later as a publisher submitting materials for approval. This experience showed me how important it is to have professional mathematicians involved in the approval process. They add an important dimension to the understanding of the materials, and they play a significant role in the process."

Dr. Gamelin became deeply involved in mathematics education in the late 1990s, initially as a math specialist in connection with the California approval process for school math textbooks, and then in 1999 as faculty advisor to the California Mathematics Project at UCLA. Dr. Gamelin continues to be involved in writing projects for the Center for Mathematics and Teaching, whose current focus is on middle school mathematics. He is also a primary author of textbooks that have gone through the California approval process.

Dr. Gamelin graduated from Yale in 1960 and obtained his Ph.D. at Berkeley in 1963. During this time, he took courses from Calvin Moore and Shiing-Shen Chern, who later founded MSRI. After spending several years at MIT and in Argentina, Dr. Gamelin joined the UCLA Mathematics Department in 1968, where his career spanned 40 years before he retired. The main focus of his research is the area between functional analysis and complex analysis. He authored a couple of research monographs on function algebras and a textbook on complex analysis. He also co-authored a textbook on topology and a monograph on complex dynamical systems.

New Giving Societies to Recognize MSRI Donors

Heike Friedman

If you are a loyal Emissary reader, you may notice that something important is missing in this issue: in the past, we used the fall edition to thank our donors for their support. We have to decided to change this practice; going forward, we will publish the list of our supporters in the spring issue. This will allow us to thank all our donors who have made a gift to MSRI in the prior full calendar year.

This is not the only change to our annual giving program; eleven years after its inauguration, we've given our Archimedes Society a face lift: the giving levels have been updated, and our community of supporters at the Museion level ($5,040+) has grown to a size that warranted its own Society with three tiered giving levels.

We are establishing a third, new giving society, named for the German mathematician David Hilbert (1862–1943), who contributed substantially to the establishment of the formalistic foundations of mathematics. Donors who make a gift of $100,000+ during a calendar year will become members of the Hilbert Society for one year; friends who have supported MSRI with a cumulative giving of $1 million will become lifetime members. The Hilbert Society will be inaugurated in the spring with a special celebration.

One third of MSRI's operating budget comes from private support from foundations and from individual donors. This support allows us to add to our core program — to run an applied mathematics workshop, for example, like "Breaking the Neural Code"; it also enables us to lend our expertise to impact K-12 education; and it makes it possible for us to create public events and initiatives.

For more information about on how to support MSRI, please visit msri.org/support-msri.

Puzzles Column

Elwyn Berlekamp and Joe P. Buhler

1. Two non-perpendicular intersecting lines m and n are drawn in the plane. You are allowed to draw a new circle and a new line k (using a compass and straightedge respectively). Find a way to draw these new two objects so that the line k is perpendicular to the line n.

Comment: We heard this problem from Gregory Galperin.

2. A certain lock has three wheels, A, B, C, each of which can be set to four positions. The lock will open when any two of the wheels point at the correct number. So it is easy to open the lock in 16 tries by just trying all the possibilities for the A and B wheels. Show how it can be done in a much smaller number of tries.

Comment: We heard this problem from Stan Wagon.

3. Alice and Bob play the following game. A referee is assigned to each player, and Alice and Bob are taken into separate rooms by their respective referees. Each referee hands an infinite bit string to the player, with Alice receiving the bit string

$$A_0\, A_1\, A_2\, \dots\, ,$$

and Bob receiving

$$B_0\, B_1\, B_2\, \dots\, ,$$

where each bit A_i or B_j is in $\{0,1\}$.

All bits are independent and uniformly random, that is, each A_i and B_j is determined by independent flips of a fair coin. (In particular, the bit strings are distinct with probability 1.)

Alice and Bob then give nonnegative integers a and b to their referees. If A_b is equal to B_a then the players win, otherwise they lose.

If this was all that there was to the game, it might seem as if Alice and Bob can do no better than guess, giving them probability $\frac{1}{2}$ of winning.

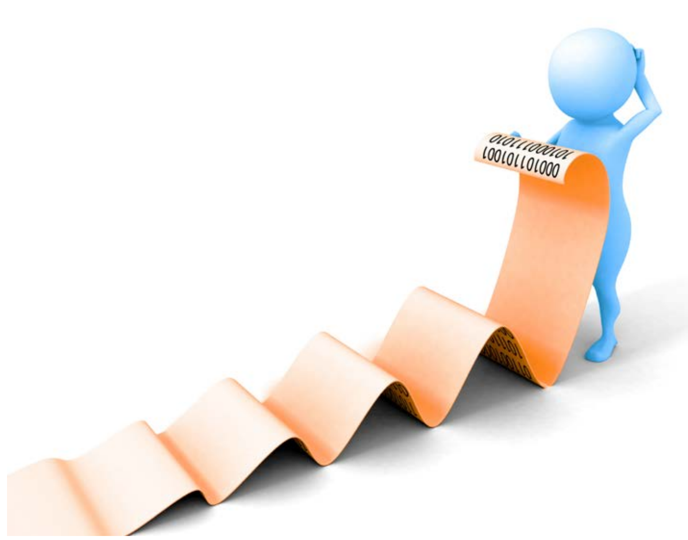
However, the game is explained to them beforehand and they are allowed to have a strategy session the night before they play. Find a strategy that has probability of success larger than $\frac{1}{2}$.

(This may sound impossible, but there's no trick involved; in fact we do not know the best possible probability.)

4. A team of n players is allowed a strategy session, after which they are taken into a room, at which point *no* further communication between them is allowed. An infinite tower of hats is placed on each of their heads; each hat is black or white, and the colors are chosen by independent flips of a fair coin.

After enough time has elapsed for each player to see all hats, except of course the ones on his or her head, a bell rings and the players are required to simultaneously (and without communication) name a positive integer.

If every player names the position of a black hat on his or her head then the team wins, otherwise they lose.



Obviously if they all guess (that is, they waste their strategy session) the probability of success is $1/2^n$. Find a strategy that increases this to something on the order of c/n for some constant c.

5. Modify the preceding problem in the following way: the players win if and only if they point to (that is, name the position of) the *lowest* black hat on their heads.

a) Find a strategy with the success probability on the order of $c/\log(n)$ for some constant c.

b) What is the best asymptotic value of c?

Comments on problems 3, 4, 5: The astute reader will notice that that 5a solves problem 4, and then some. It is striking that the winning probability of a naive strategy is $1/2^n$, a nontrivial strategy has c/n, and an even better strategy for a more difficult problem has $c/\log(n)$. We do not know if this is best possible for problem 4. This is a really surprising problem (with a strong whiff of paradox; most people rebel against the asserted winning probabilities when they first hear them). It was invented several years ago by Lionel Levine (at Cornell). The especially astute reader will note that problem 3 is closely related to problem 4 for two people.

There's quite a bit that can be said about this problem, and some comments about what is known and not known will be posted in due course at http://www.msri.org/web/msri/pages/16.

6. What is the smallest number of cages in a 6×6 Ken-ken problem such that there is a unique solution?

Comment: As most readers are probably aware, an $n \times n$ Ken-ken puzzle is an n by n grid of squares that are partitioned into "cages" consisting of connected squares. Each cage is annotated with one of the four basic arithmetic operations and a "target" integer. The grid must be filled in with integers between 1 and n such that no integer occurs twice in a row or column, and the given target results from applying the arithmetic operation to the integers in that cage. (Subtraction/division occur only for cages with two squares, and can be applied to the integers in either order, as desired.) Noam Elkies gave us this problem.

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