Fall 2011

EMISSARY

Mathematical Sciences Research Institute

www.msri.org

MSRI Founder Honored

A word from Director Robert Bryant

Mercedes Bouchard

Wei Li Wang's rendition of S.-S. Chern

The fall semester here at MSRI has been an exciting one. In addition to our scientific program, Quantitative Geometry, which has attracted an unusually diverse group of mathematicians from fields such as classical geometry, group theory, computer science, and computational complexity, we've had a variety of other fascinating activities and workshops. A big event in our history is the celebration of the 100th anniversary of the birth of our founding director, S.-S. Chern, about which I will say more below.

In September, MSRI and the CME Group in Chicago awarded the 2011 CME Group-MSRI prize to Thomas J. Sargent, the W. R. Berkley Professor of Economics at NYU, for his innovative use of mathematics in macroeconomics and rational expectation theory. The award ceremony was held at the CME on September 26 and included a lively panel discussion on the incredibly timely topic of sovereign debt featuring a panel of international experts. A video of this discussion is available at the CME Group web site: See http://tinyurl.com/msri-cme.

Our prize selection committee, chaired by David M. Kreps, a former recipient of the prize, was gratified a couple of weeks later to learn that Professor Sargent has been awarded the 2011 Nobel prize in Economics. Our congratulations go to Professor Sargent, whose work has especially emphasized the importance of mathematics in economics models and their interpretation.

October began with another in our series of Conversations with Robert Osserman, this time with fabled mathematician and hedge fund manager James H. (Jim) Simons. The October 3 event, entitled *From Math to Market and Back*, was held at the Berkeley City College. Bob and Jim talked for over an hour about Jim's early career as a mathematician (including his collaboration with S.-S. Chern, which resulted in their discovery of the Chern–Simons

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A Word from the Director

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functional), his interest and eventual concentration on finance, and his return to mathematics in recent years. The audience was thoroughly engaged, and a lively period of questions and answers followed. The conversation (which was filmed) should be available on our web site very soon, and I think you'll enjoy it.

On October 30, we have a special event: Since this is the 100th anniversary of the birth of S.-S. Chern, we are marking the occasion with the dedication of a statue of Professor Chern installed in the newly renovated entry to Chern Hall and the premiere of a film, *Taking the Long View: The Life of S.-S. Chern.* The statue, which was commissioned with the support of the Simons Foundation, is by the leading Chinese-American sculptor Wei Li ('Willy') Wang, whose work graces major museums in both China and the US. The film, which contains much archival material from Chern's life and features extensive interviews with many of Chern's family, friends, and collaborators, is by George Csicsery, famous for his film biographies of other mathematical figures, such as Paul Erdős and Julia Robinson, and his interviews with a wide range of mathematicians.

This special evening is to be followed immediately by the Chern Centennial Conference, a weeklong conference celebrating Chern's work in his beloved geometry with a series of lectures on recent developments in the field by its current leaders. We are delighted to be able to bring together so many of the people who have been inspired by Chern's life, and we are pleased to be able to thank the Simons Foundation for its support of the conference. In case you weren't able to join us for the conference, we expect to be posting the videos of the lectures this fall, and I hope that you'll take the opportunity to sample some of them.

MSRI relies on the generous donation of time and talent of many in the mathematical community, for which we are most grateful. When members of our MSRI family receive special recognition, we are most pleased to be able to congratulate them. This fall has seen a number of such recognitions: Professor Srinivasa (Raghu) Varadhan of NYU, who serves on our Scientific Advisory Committee, received the 2011 National Medal of Science, as did Richard Tapia of Rice University, who has worked with and still inspires our Human Resources Advisory Committee in our efforts to promote opportunities for underserved minorities in the mathematical sciences. In addition, Jennifer T. Chayes, who serves on our Board of Trustees, was honored by the Girl Scouts of America for her work in encouraging women to pursue careers in mathematics. We offer congratulations to all of them for this recognition of their tremendous service to the mathematical community and society.

You will see some new faces here at MSRI on your next visit: This summer, Chris Marshall joined us as our new Program Manager and Jasmine Partida joined us as our new Workshop Coordinator. We also have two new interns, Cecille Reyes and Mick Santos, who will be helping in various ways on a part-time basis. Finally, you will notice some new people providing our computing and IT support. This year, we hired Uptime, a company headed by Bob Hoover that provides IT support to many small businesses and non-profits in the Bay Area, to provide these services for MSRI. It is a pleasure to welcome all of these people to our MSRI team, and I hope that you'll stop by and say hello to them on your next visit.

There is another change coming, I would like to encourage you to help get the word out: Next summer will mark the end of three years that we have been fortunate enough to have Dave Auckly as our Associate Director. Dave has done a wonderful job with Math Circles, our mathematics education activities, enhancing our diversity efforts, and representing MSRI in many other ways. However, Dave now needs to return to his position at Kansas State University and MSRI is actively looking for our next Associate Director. Information about the position can be found below and on our web site. If you know of someone who would be right for this position, please encourage them to apply. We will be considering applications starting on December 1.

If you are coming to Boston for the Joint Meetings, please come and see us at the All-Institutes reception on Wednesday, January 4 (see back page of this issue). In the meantime, please accept my best wishes for the end-of-the-year season.

Associate Director Position Open

MSRI's outreach activities — to other sciences, to the public, and to various efforts in education — are central to its role in the mathematical community. The Associate Director oversees these outreach projects, which include workshops, Math Circles coordination, and corporate-affiliate activities. He or she has the responsibility to administer the current projects as well as oversee the creation of new ones (when appropriate). The Associate Director reports to the Deputy Director, works with the Director and the Deputy Director on all phases of Institute activity, and helps to formulate Institute policy.

The Associate Director should have a broad understanding of mathematical culture, demonstrate an interest in outreach activities, and have substantial administrative experience.

An application must include a curriculum vitae, a list of at least three references, and a statement of interest, explaining why the applicant is interested in coming to MSRI and what he or she would bring to the position. For more information, or to submit an application, contact ADsearch@msri.org.

MSRI Undergraduate Program Turns Five

David Auckly and MSRI-UP directors

Summer 2011 marked the fifth year of the MSRI undergraduate program (MSRI-UP). This program is much more than a traditional REU. It is a comprehensive program that identifies undergraduate students with an interest in mathematics and constructs a path that can take them from a math major, through graduate school, to a position where a research career in mathematics is a viable option. The MSRI-UP is designed to provide the opportunity and the long-term support that talented undergraduate students need in order to have access to research careers in science and mathematics.

The focus of the program is on US students and underrepresented minorities in particular. The reason for this focus is that there has been a downward trend in the percentage of mathematics Ph.D.s awarded to U.S. citizens and permanent residents over the last decade, and tapping talent from underrepresented groups may be one way to change this trend. It is reported that in 2006 this number was 46% while in 1995, this figure was 65% and it was close to 70% in the early 1980s.

The proportion of the US population consisting of African Americans, Hispanic/Latinos, and Native Americans is expected to grow to 42% by the year 2050. Today the proportion of US citizens awarded Ph.D.'s from these groups remains less than 7%. There is an obvious mismatch between this figure and the fact that the same groups comprise 30% of the total US population.

The idea of having an undergraduate research program hosted by MSRI developed as a result of a presentation by Prof. Ricardo Cortez of Tulane University at MSRI in the spring of 2006. Cortez spoke at the meeting titled *Raising the floor: Progress and setbacks in the struggle for quality mathematics education for all*. He spoke about his experience about what works and what does not work with regards to having successful mathematics research experiences for undergraduates with an emphasis on the inclusion of underrepresented minorities. David Eisenbud (MSRI director in 2006) approached Cortez after the talk to ask whether "something like this" could be done at MSRI. Cortez, Prof. Suzanne Weekes of Worcester Polytechnic Institute (WPI), Prof. Duane Cooper of Morehouse College, Prof. Herbert Medina of Loyola Marymount College, and Prof. Ivelisse Rubio of the University of Puerto Rico at Rio Piedras founded MSRI-UP. To date, just over 80 students can count themselves alums of MSRI-UP. The research director changes from year to year as does the research theme.

Running a successful program to recruit and track students from historically underrepresented groups into careers in mathematics is a very formidable task. To distribute this workload, MSRI-UP has a team of five directors that run all aspects of the program. Every year one of these directors serves as the onsite director, and the others serve as advisors. Since the commitment is to have one intense summer every five years, the burden is not too great to sustain. The original five founders serve as the directors. In addition to the onsite directors, each program has a faculty member designated as the research leader, one postdoctoral fellow and two graduate student assistants. The directors, research leaders, and topics from the first five programs are as follows:

2007	Ricardo Cortez	Juan Meza	Computational Math
2008	Ivelisse Rubio	Victor Moll	Experimental Math
2009	Herbert Medina	John Little	Coding Theory
2010	Duane Cooper	Edray Goins	Elliptic Curves
2011	Suzanne Weekes	Marcel Blais	Mathematical Finance

The success of program students and directors has already begun to be recognized at national forums. Indeed, three of the four MSRI-UP posters presented at the 2011 Joint Math Meetings were awarded prizes. The fourth was one of three prize-winning groups at the 2010 SACNAS meeting. In 2010, Prof. Rubio received the Dr. Etta Z. Falconer Award for Mentoring and Commitment to Diversity and was also selected to the US National Committee for Mathematics. In 2010, Professor Cortez was honored with the SACNAS Distinguished Undergraduate Institution Mentor Award in recognition of his contribution to minority education activities at the local and national level.

In 2011 eighteen undergraduates, two professors, one post-doc, and two graduate assistants spent 6 intense weeks in the beautiful hills of Berkeley, California studying mathematics and doing original mathematics research. Prof. Weekes was the on-site director of the 2011 program and Prof. Marcel Blais, also from WPI, was the research director and introduced the students to the rich field of mathematical finance.

For the first two weeks of the 2011 program, Prof. Blais delivered to the students what amounted to more than half of an introductory graduate course in financial math, and the students did tons of homework and studied with the help of the graduate assistants, who lived in the UC Berkeley dormitories with the undergraduates. After just week one of this “mathematical hazing”, as one student put it, the students already felt bonded to each other and to the MSRI-UP staff. Over the weeks, the students quickly learned Matlab, Latex, and Beamer, made several presentations within the 6 weeks, attended many colloquia, took many trips up and down the Berkeley hills — oh, and, yes, they did some original research! The abstracts of their research papers can be found on the MSRI-UP 2011 website.

The 2011 MSRI-UP's line-up of colloquium speakers included Prof. Talithia Williams of Harvey Mudd College, Prof. Joseph Teran of UCLA, and Nobel Laureate Prof. Myron Scholes. Given that the Black–Scholes model is one of the fundamental approaches to pricing financial derivatives, it was a special treat to spend time with Scholes.

Weekend trips included spending the day at the Santa Cruz Beach Boardwalk, touring San Francisco, kayaking in Oakland, going to the Monterey Bay Aquarium, and a Fourth of July Oakland A's game. After the 6 weeks of the 2011 MSRI-UP, students left more mathematically sophisticated than when they came in, more determined to go to graduate school than when they arrived, more knowledgeable about applying to graduate school and what to expect once they start, more exhausted than when they showed up, and more exhilarated than ever! Comments from the students attest to this:

> When I realized that we were doing graduate level work and I could successfully understand the concepts and apply them, I grew in confidence and felt that I could manage graduate school.

> Before this program I was unsure about grad school and I did not know about applied mathematics. Now, I am looking into applied math programs for grad school and I am positive that I want to go to graduate school. This program gave me the confidence and assurance I needed to know that I can go to grad school for math. Thank you!

In addition to the rich and intense mathematical work, students are introduced to the mathematical community and to a network of peers and mentors by presenting at national conferences, such as the SACNAS National Conference, NAM's MathFest, and the Joint Mathematics Meeting. This successful partnership between MSRI and the MSRI-UP Directors was praised by a site-visit team in September of 2009:

> The team notes that this is a dream come true to address the serious national issue of the need to increase the applicant pool of American female and minority Ph.D. students in mathematics.

The summer 2012 MSRI-UP program will be led by Prof. Ricardo Cortez. The research director will be Prof. Matthias Beck of San Francisco State University and the research theme will be Enumerative Combinatorics. Applications will be taken from December on: see the link below.

http://www.msri.org/web/msri/education/for-undergraduates/msri-up

MSRI - UP

Chris Marshall

Jasmine Partida

Cecille Reyes

Christine Marshall, our new Program Manager, comes to MSRI with over 20 years of management experience in the non-profit sector. As the former General Manager for a professional theatre company in the Bay Area, she has a wealth of experience organizing multi-faceted events. Chris enjoys working with a broad range of personalities and is excited to bring her organizational and planning skills to MSRI in her position as Program Manager.

Jasmine Partida is an artist with an array of administrative experience, having previously worked at Opera San Jose and Ryan Young Interior Design Firm. She is a Bay Area native who looks forward to utilizing her creativity and efficiency in her new role as Workshop Coordinator. Jasmine loves working with people, which makes her a perfect fit for this position at MSRI.

Cecille Reyes is a senior at the University of California, Berkeley, majoring in economics and accounting. As an MSRI Program Analyst Intern, she is putting her studies to good use. Cecille assists with the preparation and financial budgeting of MSRI's workshops, programs, and summer graduate schools.

Mick Santos is finishing up his studies at UC Berkeley in integrative biology. He plans to pursue a career in nursing, public health, or education. As an MSRI Program Intern, Mick continues to foster his passion for education and serving the community by assisting in the coordination of MSRI's programs and workshops.

Mick Santos

MSRI computers are now administrated by **Uptime USA**, a San Francisco firm that has been operating the computer departments for small to medium organizations in the Bay Area for twenty years. Uptime manages MSRI's servers, networks, Internet, and email systems, allowing MSRI staff and members to focus on mathematics. The Uptime team is available full-time onsite and remotely to support MSRI.

The Dehn Function and the Origins of Quantitative Geometry

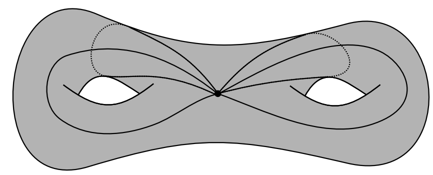
Robert Young

We're halfway through the Fall 2011 Quantitative Geometry program at MSRI. The first question I get when I mention the program is "What is quantitative geometry?" The program is indeed broad. It includes people from many different disciplines, including topologists, geometers, probabilists, analysts, and computer scientists. Its title, "Quantitative Geometry", covers everything from random walks on groups to bounds on the Bernoulli distribution on the hypercube to the study of Lipschitz functions on $\mathbb{R}^n$.

The program is too big for me to describe fully, but I can at least describe my view on the subject. For me, much of quantitative geometry starts by taking a theorem which says that something exists, and asking "How big is it?" or "How many are there?" or "How fast does it grow?" In other areas of mathematics, this approach leads to things like complexity theory or the Prime Number Theorem. In quantitative geometry, one of the best examples of this approach is the Dehn function.

The Dehn function quantifies simple connectivity: in a simply connected space, every closed curve is the boundary of some disc, and the Dehn function describes the size of that disc. It originated from a combinatorial problem. Max Dehn wanted an algorithm to decide whether a product of generators (a word) in the fundamental group of a closed surface represents the identity; this question is called the word problem. His insight was that this problem is not just a combinatorial problem but also a geometric problem: words in the fundamental group correspond to paths in the universal cover of the surface, and if a word represents the identity, it corresponds to a closed curve, and thus to the boundary of some disc.

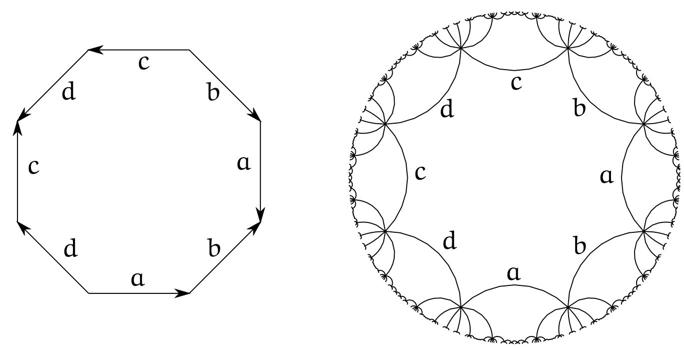
To see how this works, consider Dehn's original example, the surface of genus g. We can build the surface of genus g by gluing a disc to a wedge of $2g$ circles, like this:



Thus the fundamental group has a generating set with one generator for each circle. A product of generators of $\pi_1(\Sigma_g)$ corresponds to a closed path in that wedge, and since the boundary curve of the disc is homotopically trivial, the corresponding word is a relation in the group. In fact,

$$\pi_1(\Sigma_g) = \langle a_1, \dots, a_g, b_1, \dots, b_g \mid a_1 b_1 a_1^{-1} b_1^{-1} \dots a_g b_g a_g^{-1} b_g^{-1} = 1 \rangle.$$

Since the surface is formed by gluing the edges of a $4g$-gon (see figure below), the universal cover of the surface is a tiling of the hyperbolic plane by $4g$-gons. Dehn used this tiling in an algorithm for solving the word problem. The basic idea of the algorithm is that if a word contains more than half of a relator, one can shorten it



The universal cover of a surface is a hyperbolic plane tiling.

Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the Scientific Advisory Committee with a copy to proposals@msri.org. For detailed information, please see the website www.msri.org.

Thematic Programs

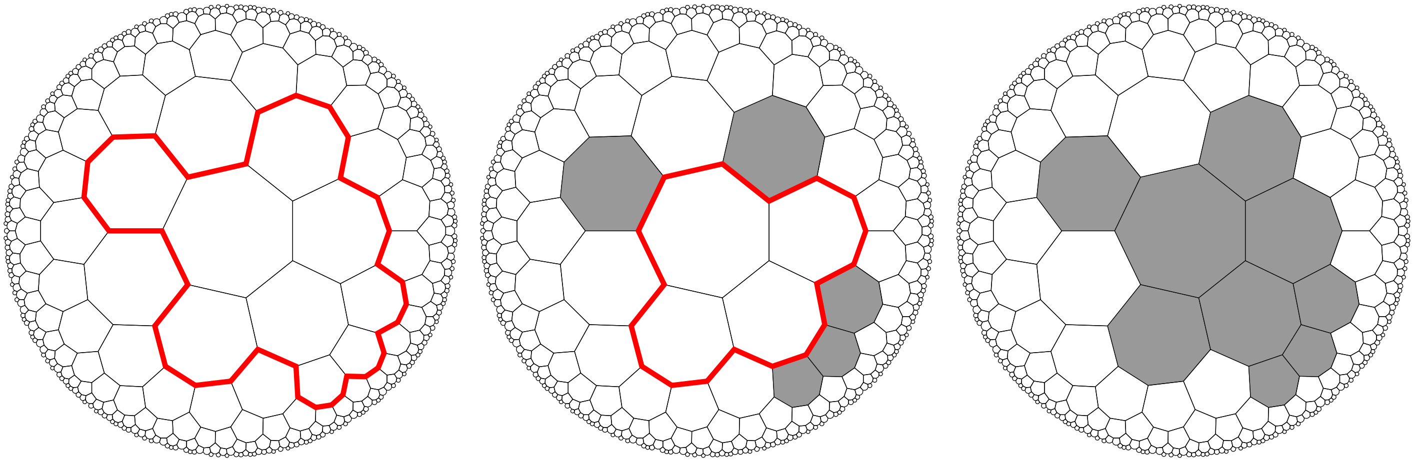
Letters of intent and proposals for semester or year long programs at the Mathematical Sciences Research Institute (MSRI) are considered in the fall and winter each year, and should be submitted preferably by **October 1** or **December 30**. Organizers are advised that a lead time of several years is required, and are encouraged to submit a letter of intent prior to preparing a pre-proposal. For complete details see http://tinyurl.com/msri-progprop.

Hot Topics workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals for such workshops should be submitted by **October 31** or **December 30**. See http://tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes four 2-week long summer graduate workshops, most of which are held at MSRI. To be considered for the summer of year n, proposals should be submitted by **October 31** or **December 30** of year $n-2$. See http://tinyurl.com/msri-sgw.



The negative curvature of the hyperbolic plane means that closed curves in this tiling contain over half of the boundary of one of the tiles. Moving the curve over one of these tiles shortens it, constructing a disc filling the curve.

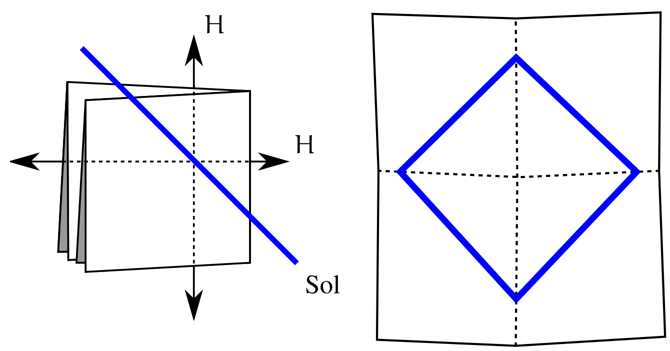
it by replacing that half with the other (shorter) half of the relator. Dehn showed that if a word represents the identity in $\pi_1(\Sigma_g)$, it must contain more than half of a relator. So, given a word, one can repeatedly shorten it by replacing large pieces of relators — if there are no large pieces left, it can't represent the identity, and if the algorithm eventually reduces the word to nothing, it must represent the identity.

Dehn's algorithm shows how the geometry of a group can affect its combinatorics. The algorithm works because of negative curvature. Words in $\pi_1(\Sigma_g)$ correspond to paths in the universal cover, and words that represent the identity element correspond to closed curves. Because the tiling is negatively curved, any closed curve must contain more than half of the boundary of a face (see figures above); but the boundary of a face is exactly the relator in the group, so any word representing the identity contains more than half of a relation.

But this sort of shortening algorithm doesn't work for all groups. Indeed, because each step in the algorithm reduces the length of the word, any word of length ℓ that represents the identity can be reduced to the identity in no more than ℓ steps. The function that counts the number of steps it takes to reduce a word of length ℓ to the identity is called the Dehn function of a group, and there are groups with very large Dehn functions, like the Baumslag–Solitar group $BS(2,1)$ (exponential Dehn function), certain one-relator groups (Dehn function larger than any tower of exponentials), and groups with unsolvable word problem (Dehn function larger than any computable function). No shortening algorithm can solve the word problem in such groups; there are words that represent the identity, but cannot be reduced to the identity without intermediate steps of uncomputably large length.

So when does the algorithm work? The answer is a beautiful example of how the combinatorics of a group can affect its geometry! Small Dehn functions are linked to negative curvature: in the 1980s, Gromov gave a geometric condition, called word-hyperbolicity, which exactly describes which groups have a linear Dehn function, and Lysenok and Cannon showed that these groups are exactly those groups in which the word problem can be solved by a shortening algorithm.

Remarkably, large Dehn functions can also come from negative curvature. For example, the 3-dimensional model geometry Sol is a space with exponentially large Dehn function — there are curves of length ℓ which bound discs of area $\sim e^{\ell}$. The reason why is that Sol is a subset of a product of hyperbolic planes, $H \times H$! In the hyperbolic plane, any short path between two points must be close to a geodesic. In $H \times H$, the same is true for discs: products of geodesics form flats, and any small filling of a curve in a flat must be close to that flat. So, to find a curve in Sol which is difficult to fill, it's enough to find a disc in $H \times H$ which only intersects Sol along its boundary (see figure below). Since that curve has a small filling which goes far from Sol, all of its small fillings must go far from Sol, and any filling *in* Sol must be large!



Sol is contained in a product of two hyperbolic planes as in this schematic figure. That product has "folded flats" which intersect Sol. When they're unfolded, these flats intersect Sol in hard-to-fill diamonds.

Similar pictures show up in subsets of other non-positively curved spaces, like lattices in semisimple groups, and what we often find is that the difficulty of filling depends on the size of flats: a subset of a rank-2 symmetric space, like Sol, often has exponential Dehn function, but there is more room in the flats in a higher-rank symmetric space, and subsets may have smaller Dehn functions. Results of Druţu, Young, and Bestvina–Eskin–Wortman have confirmed this in several cases, giving us a new view on the arrangement of flats in symmetric spaces and their geometry.

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Focus on the Scientist: Marianna Csörnyei and Jelani Nelson

Marianna Csörnyei works in geometric measure theory, especially on exceptional sets and rectifiability. Over the last thirty years, a common approach to many problems in nonlinear geometric functional analysis arose from the discovery of new classes of exceptional sets in infinite dimensional spaces. When X is infinite dimensional it is impossible to find a measure μ for which the sets of μ-measure zero form a useful class: so we need to define negligible sets by other means. The three main classes of negligible sets are the so-called Aronszajn null sets, Gaussian null sets, and cube null sets. For a while, it was widely believed that these classes were intrinsically different from one another. Marianna showed that in fact Aronszajn, Gaussian, and cube null sets coincide.

Marianna generalized Davies' theorem about covering sets by lines with the same total measure by showing that for any σ-finite Borel measure, any Borel set can be covered by a union of lines of the same measure. This is equivalent to the following statement: for any measure, one can prescribe a family of lines and then find a set which meets exactly those lines (modulo the measure). In other words, there is a "digital sundial": a set which at 12:00 casts the shadow **12:00**, which at 12:01 casts the shadow **12:01** and so on.

One version of the Gorelik principle says that a bi-uniform homeomorphism between Banach spaces cannot map the unit ball of a finite-codimension subspace into a subspace of larger codimension. Many people wondered whether the Gorelik principle holds for Lipschitz quotient maps between reflexive spaces. Marianna answered this question negatively even in the finite-dimensional case. As a consequence, it fails for Lipschitz quotient maps even from ℓ_2 to itself. The result has profound consequences in two very different branches of modern analysis: the geometry of Banach spaces and the theory of quasi-regular mappings.

In ongoing research with G. Alberti, P. Jones, and D. Preiss, Marianna has shown how product decompositions of measures can detect directionality in sets, and how this can be used to find tangent fields to null sets and nondifferentiability sets of Lipschitz functions. These results are closely related to a wide range of problems in the theory of partial differential equations and in the calculus of variations, from the structure of normal currents to the regularity of minimizers of certain variational problems.

Marianna graduated from Loránd Eötvös University in 1999 and received her Ph.D. a few months later. Since then she has held positions at University College London and the Institute for Advanced Study, and been a visiting professor at Yale University. In 2010 she was an invited speaker at the International Congress of Mathematics. Since September 2011, she has been a professor at the University of Chicago.

Jelani Nelson studies problems in theoretical computer science. Much of his work has been in the development of memory- and time-efficient *streaming* algorithms: algorithms that are able to compute some function of a data steam using only one pass. In joint work with Daniel Kane and David Woodruff in 2010, he gave the first optimal algorithm for the distinct elements problem and the first space-optimal algorithm for the p-th moment estimation problem for p less than 2. With colleagues he has also developed efficient pseudorandom generators, entropy estimation algorithms, cache-oblivious data structures, and selection and sorting algorithms given a faulty comparator.

Recent work of Jelani's includes an investigation of several problems related to dimension reduction under the Euclidean norm. A lemma of Johnson and Lindenstrauss from 1984 (the "JL-lemma") asserts that for any n points in arbitrary dimension and any $0 < \varepsilon < \frac{1}{2}$, there exists a linear map into $O(\varepsilon^{-2} \log n)$ dimensions such that all pairwise Euclidean distances are preserved up to a $1 \pm \varepsilon$ factor. This lemma has found numerous applications in computer science, including high-dimensional computational geometry, information retrieval, and numerical linear algebra. Jelani and his colleagues have improved the state of the art in several computational aspects of the implementation of the JL lemma. For example, in joint work with Daniel Kane he gave a construction of such maps that gives the fastest known embedding time when the n vectors have sufficiently low average sparsity. Jelani's work with Kane and Raghu Meka also shows how to select a small family of dimension-reducing mappings so that for any n point set, one of the mappings is guaranteed to provide a good embedding.

Jelani recently completed his Ph.D. in computer science at MIT, in the CSAIL Theory of Computation group. In January 2012 he will begin as a postdoctoral fellow jointly at the Princeton University computer science department and the Institute for Advanced Study. In mid-2013 Jelani will join the computer science faculty at Harvard University.

The Dehn Function

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So the Dehn function comes full circle, from Dehn's use of the geometry of the hyperbolic plane to produce algorithms to more recent results using algorithmic ideas to explore the geometry of symmetric spaces. This is what makes quantitative geometry so exciting to me: the hope that we can use new methods, whether algorithms, random walks, or Lipschitz embeddings, to explore familiar geometries or understand new ones.

Puzzles Column

Elwyn Berlekamp and Joe P. Buhler

1. Out of a collection of 25 horses you can choose 5 of them to race on a fixed course. The horses always run at a constant speed and their speeds are distinct. At the end of the race all you are told is the order of finish of the horses.

How many such races are necessary for you to determine (with proof) the fastest three horses, and the order of those horses?

Comment: This seems to be a folklore puzzle, and rumor has it that it was used by Facebook as part of their interview process.

2. Three identical cakes must be shared equally among five children. The kids are a bit fussy, and really like large pieces of cake. Your job is to cut the cakes into pieces that can be allocated to the children in such a way that (a) each child receives the same total amount of cake and (b) the size of the smallest piece (of all of the pieces) is as large as possible.

One obvious solution is to divide each cake in fifths and give each child three of them. Can you do better?

Comment: Jeremy Copeland found this in some teaching materials for gifted math students.

3. Let h, r, and f be positive integers with $h \geq r \geq 2$. Generalizing the first problem, you have h horses, and you can race r of them at a time; your goal is to determine (with proof) the f fastest horses, and the order of those f horses.

Let $N(h, r, f)$ be the smallest number of races that are necessary to accomplish this goal. Show that

$$N(h, r, f) \geq \frac{h-1}{r-1}.$$

Comment: Problem 1 was brought to our attention by Stan Wagon and Stephen Morris as part of their work on generalizations, and Richard Stong told us about Problem 3. A reader searching for an extra challenge might want to show that the bound in this problem is almost optimal: when r and f are fixed as h grows to infinity,

$$N(h, r, f) = h/(r-1) + O(\log h),$$

where the constant implicit in the $O(\,)$ notation depends on r and f. This means that the work required to find the top f horses is only slightly more than the work required to find the single fastest horse. This apparently first appeared in an Olympiad training exam in India, and subsequently in *Crux Mathematicorum* as well as Stan Wagon's Macalester *Problem of the Week* online column.

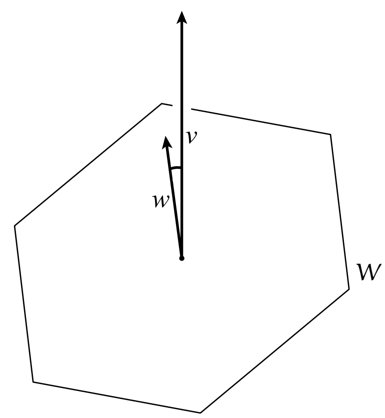
4. Say that a row of Pascal's triangle is *good* if it contains four distinct values a, b, c, d such that $a = 2b$ and $c = 2d$.

(a) Find a good row.

(b) Prove that there are infinitely many good rows.

Comment: This problem was on the 2011 BAMO exam.

5. Fix a positive integer n. Let V be the $(n-1)$-dimensional subspace of $\mathbb{R}^n$ consisting of those n-tuples that sum to 0. For which n does there exist a hyperplane in V (i.e., an $(n-2)$-dimensional subspace of $\mathbb{R}^n$ consisting of zero-sum vectors) that makes the same angle with each coordinate axis in $\mathbb{R}^n$?



In a vector space, the angle between a vector v and a vector subspace W of arbitrary dimension is defined as the minimum angle between v and a vector w in W.

Comment: Gregory Galperin, who has provided some sterling puzzles to our column in the past, came up with (a more general version of) this problem, as well as the difficult and delightful question that follows.

6. You are allowed to transform positive integers n in the following way. Write n in base 2. Write plus signs between the bits at will (at most one per position), and then perform the indicated additions of binary numbers. For example, $123_{10} = 1111011$ can get $+$ signs after the second, third and fifth bits to become $11+1+10+11 = 9_{10}$; or it can get $+$ signs between all the bits to become $1+1+1+1+0+1+1 = 6_{10}$; and so on.

Prove that it is possible to reduce arbitrary positive integers to 1 in a bounded number of steps. That is, there is an absolute constant C such that for any n there is a sequence of at most C transformations that starts with n and ends at 1.

Comment: The best possible bound is a real shocker.

MSRI meets INdAM: Toric Varieties at Cortona

In partnership with INdAM (Istituto Nazionale di Alta Matematica) and the SMI (Scuola Matematica Interuniversitaria), MSRI sponsored a summer graduate workshop on toric varieties in Cortona, Italy, from July 18 through 29, 2011. INdAM has been organizing summer programs for graduate students for over fifty years, and MSRI for over twenty years, but this marks the first collaboration between the two institutes. The workshop was truly international in flavor: the thirty six participants hailed from eight different countries, and represented universities in Italy, Korea, the Netherlands, Romania, Russia, Spain, the UK and the USA. The organizing team consisted of Giorgio Patrizio (Firenze), Sandro Verra (Roma Tre), David Cox (Amherst) and Hal Schenck (Illinois), aided by MSRI staff (Vanessa Kääb-Sanyal and Chris Marshall), local staff (Silvana Boscherini) and able scientific assistants Andrey Novoseltsev and Damiano Testa.

Toric varieties are algebraic varieties defined by combinatorial data, and there is a wonderful interplay between algebra, combinatorics, and geometry involved in their study. Many of the key concepts of abstract algebraic geometry (for example, constructing a variety by gluing affine pieces) have very concrete interpretations in the toric case, making toric varieties an ideal tool for introducing students to abstruse concepts. Participants were assumed to know the first two chapters of *Algebraic Geometry*, by Robin Hartshorne. When lectures touched on subjects (reflexive sheaves, spectral sequences) not covered in these two chapters, there was a supplemental evening lecture that summarized the material. In addition to the main goal of teaching the students toric geometry (which for us meant not simply listening to lectures, but working lots of problems), we also wanted students to gain experience making presentations in public. To accomplish all this, the workshop was structured as follows:

• Morning lectures. Each morning, there were a pair of one hour lectures by Cox and Schenck, punctuated by a break for expresso. Lectures were based on the book *Toric varieties* by Cox, Little, and Schenck.

• Afternoon problem sessions. After lunch, several "suites" of problems from the book were suggested for participants to work on. Students then split into groups. Typically there were five or six groups, with membership varying from day to day. Andrey, Damiano, David and Hal circulated among the groups while the students worked problems.

• Student presentations. After working on the problems for about three hours, one student from each group would give a 15 minute presentation on one of the problems. By the end of the workshop, every student had given such a presentation.

• Background lectures. On four evenings, there were supplemental lectures on topics not covered in a typical algebraic geometry class.

While students were assumed to know the first two chapters of Hartshorne's *Algebraic Geometry*, in fact students had a very wide range of backgrounds. This turned out to be no obstacle, as the problem selection always contained some which were concrete and computational in flavor; stronger students would also help in filling in background during the problem sessions. The

Damiano Testa

The problem sessions were an important component of the workshop. The grand architecture of the Palazzone proved to be an inspiration rather than a distraction.

final afternoon of the workshop substituted a pair of research talks: Andrey Novoseltsev spoke on Hodge numbers arising in toric mirror symmetry and Damiano Testa on Cox rings of Del Pezzo surfaces. David and Hal closed the workshop by outlining a number of open problems in toric geometry.

The workshop was held at Il Palazzone, an elegant Renaissance villa set on a hillside overlooking the Val di Chiana, built in the early 16th century by Cardinal Silvio Passerini, and donated in 1968 to the Scuola Normale Superiore di Pisa by a descendant of his. The Palazzone is a wonderful place to do mathematics: large, bright rooms, many decorated with magnificent frescos and paintings. Cortona itself is a classical Tuscan walled hill city, located just a short walk from the Palazzone. Everyone ate meals together, with lunch served in the Palazzone, and dinner in a nearby hotel (formerly a monastery) where most participants stayed. On a typical evening, after dinner participants would stroll into town and enjoy a cappuccino or grappa at the outdoor cafes. Overall, participants felt that the INdAM-MSRI collaboration was a great success: as one student remarked, "the program was excellent; I had an incredible time and learned a great deal".

Forthcoming Workshops

Most of these workshops are offered under the auspices of one of the current programs. For more information, please check out http://www.msri.org/web/msri/scientific/workshops.

October 30 to November 4, 2011: *Chern Centennial Conference*, organized by Robert Bryant and Yiming Long (co-chairs), Hélène Barcelo, May Chu, and Lei Fu

November 19, 2011: *Bay Area Differential Geometry Seminar (BADGS)*, organized by David Bao, Robert Bryant, Joel Hass, David Hoffman (chair), Rafe Mazzeo, and Richard Montgomery

December 5 to 9, 2011: *Quantitative Geometry in Computer Science*, organized by Irit Dinur, Subhash Khot, Manor Mendel (chair), Assaf Naor, and Alistair Sinclair

January 12 to 13, 2012: *Connections for Women: Discrete Lattice Models in Mathematics, Physics, and Computing*, organized by Beatrice de Tiliere, Dana Randall (chair), and Chris Soteros

January 2012 to 20, 2012: *Introductory Workshop: Lattice Models and Combinatorics*, organized by Cédric Boutillier, Tony Guttmann (chair), Christian Krattenthaler, Nicolai Reshetikhin, and David Wilson

February 6 to 10, 2012: *Hot Topics: Thin Groups and Superstrong Approximation*, organized by Emmanuel Breuillard (chair), Alexander Gamburd, Jordan Ellenberg, Emmanuel Kowalski, and Hee Oh

February 20 to 24, 2012: *Percolation and Interacting Systems*, organized by Geoffrey R. Grimmett, Eyal Lubetzky (chair), Jeffrey Steif, and Maria E. Vares

March 12 to 14, 2012: *Spring Opportunities*, organized by David Auckly, Philip Kutzko, Trachette Jackson, and Robert Megginson

This first workshop in a series addresses the professional advancement of underrepresented minorities in the mathematical sciences. It will include an introduction to mathematics represented in the MSRI research programs aimed at faculty in minority serving and primarily undergraduate institutions. Anyone who will be seeking employment in mathematics within the next couple of years would benefit from attending this workshop.

March 21 to 23, 2012: *Critical Issues in Mathematics Education 2012: Teacher education in view of the Common Core*, organized by Dave Auckly, Hyman Bass, Amy Cohen-Corwin, and William McCallum

March 26 to 30, 2012: *Statistical Mechanics and Conformal Invariance*, organized by Philippe Di Francesco (chair), Andrei Okounkov, Steffen Rohde, and Scott Sheffield

April 30 to May 4, 2012: *Random Walks and Random Media*, organized by Noam Berger, Nina Gantert, Andrea Montanari, Alain-Sol Sznitman, and Ofer Zeitouni (chair)

MSRI
Mathematical Sciences Research Institute
17 Gauss Way, Berkeley CA 94720-5070
510.642.0143 • FAX 510.642.8609 • www.msri.org

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Phone area code 510. Add @*msri.org* to email addresses.

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